PHYSICS INDUCTION
An institute of Science & Mathematics

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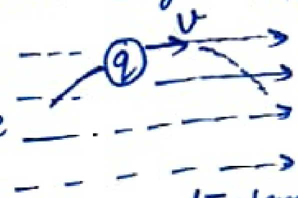
IB PHYSICS (HL) : NOTES : SPACE, TIME AND MOTION : KINEMATICS (Projectile Motion)

PROJECTILE MOTION : We consider single constant Force (always Vertical).

$F \rightarrow$ due to weight of the body. (parabolic path)

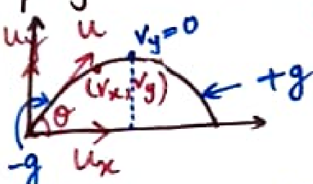


(External Electric Field)

Motion of q in uniform Electric field - Parabolic path.

Consider a situation, where a projectile is launched at an angle, θ to the horizontal with velocity, u

The components of the initial velocity vector are: $u_x = u\cos\theta$, $u_y = u\sin\theta$

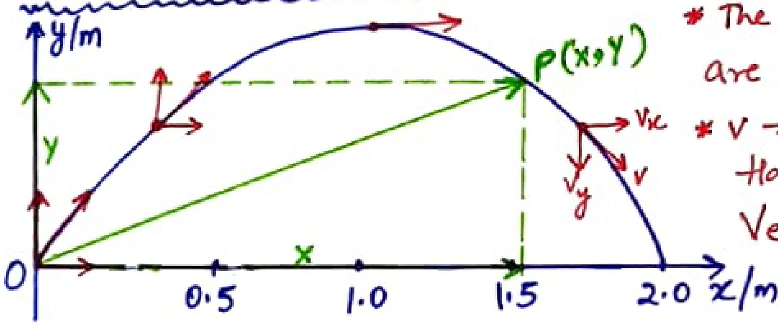


At some later time, t:

$v_x = u\cos\theta$ ($\because$ zero acceleration along x-axis)

$v_y = u\sin\theta - gt$ ($\because v = u + at$ Here, $v = v_y$, $u_y = u\sin\theta$, $a = -g$)

THE POSITION VECTOR, $\vec{r}$ – The position of the projectile is determined by $\vec{r}$ (From Origin)

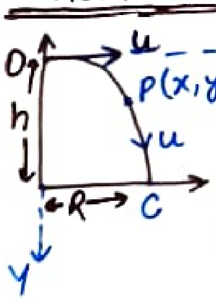


* The Velocity vectors for projectile motion are tangents to the parabolic path.
* $v \rightarrow$ combined effects of 2 velocities
  Horizontal $(v_x) \rightarrow$ constant $\rightarrow$ No Horizontal Force, $a_x = 0$
  Vertical $(v_y) \rightarrow$ Uniformly changing (↑ or ↓) due to g, $a_y = g$
* v_x & $v_y \rightarrow$ Independent of each other

Assumptions: No Frictional air resistance, No effect of Earth's rotation, Acc. due to gravity, $g \rightarrow$ constant

HORIZONTAL PROJECTION:- At $t=0$, $x_0 = 0$, $y_0 = 0$, $a_x = 0$ ($\because F_x = 0$), $a_y = g$

$u_x = u$, $u_y = 0$



Along x-axis: $x = x_0 + u_x t + \frac{1}{2} a_x t^2 \Rightarrow \boxed{x = ut}$ or $\boxed{t = x/u}$

Along Y-axis: $y = y_0 + u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = \frac{1}{2} g t^2 = \frac{g}{2}\left(\frac{x}{u}\right)^2 = \frac{1}{2} g \frac{x^2}{u^2} = kx^2$ ← Parabolic

Time of flight $(O \rightarrow C)$: $y = h$, $t = T$

$y = y_0 + u_y t + \frac{1}{2} a_y t^2 \Rightarrow h = 0 + 0 + \frac{1}{2} g T^2 \Rightarrow \boxed{T = \sqrt{\frac{2h}{g}}}$

Horizontal Range: $x = R$, $t = T = \sqrt{\frac{2h}{g}}$, $u_x = u$

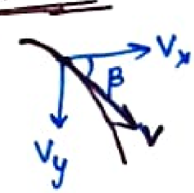
$x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$\Rightarrow R = 0 + u\sqrt{\frac{2h}{g}} + 0 \Rightarrow \boxed{R = u\sqrt{\frac{2h}{g}}}$

Velocity at an instant:

$v_x = u_x = u$

$v_y = u_y + a_y t$
$= 0 + gt = gt$



$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$

$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}$

$\beta = \tan^{-1}(gt/u)$

NOTE:
* at highest point $v \perp a$, $v_y = 0$ only v_x exists.
* T, R, H → Independent of mass
* Final velocity with which the object strikes is same as the velocity of projection.

ANGULAR PROJECTION :-

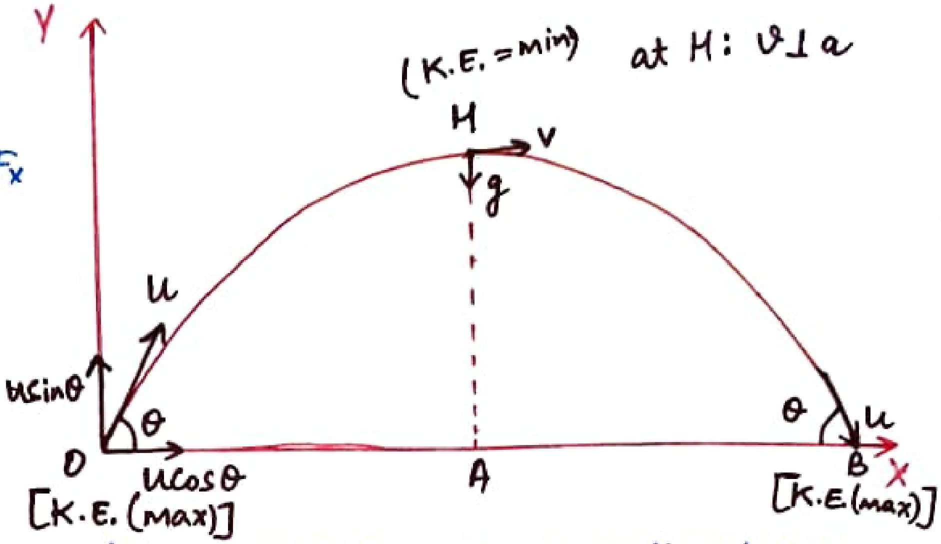
At $t=0$, $x_0=0$, $y_0=0$

$u \rightarrow u\cos\theta$ (Uniform). no F_x

$u \rightarrow u\sin\theta$

Horizontal disp : $OB = R$ due to $u\cos\theta$

Vertical displacement $AH = H$: due to $u\sin\theta$



Along X-axis :- $u_x = u\cos\theta$, $a_x = 0$

$x = x_0 + u_x t + \frac{1}{2}a_x t^2$

$x = (u\cos\theta)t \Rightarrow t = x/u\cos\theta$

Along Y-axis: $a_y = -g$, $y_0 = 0$, $u_y = u\sin\theta$

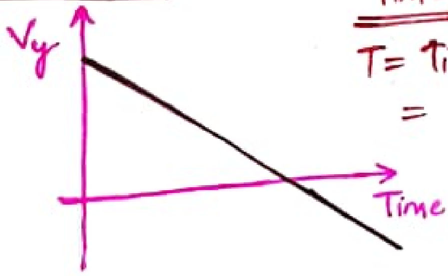
$y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$= (u\sin\theta)t - \frac{1}{2}gt^2$

$= (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

Horizontal direct^n	Vertical direction
$V_x = u\cos\theta$	$V_y = u\sin\theta - gt$
$x = (u\cos\theta)t$	$y = (u\sin\theta)t - \frac{1}{2}gt^2$

$\Rightarrow \boxed{y = x\tan\theta - \left(\frac{1}{2}\frac{g}{u^2\cos^2\theta}\right)\cdot x^2}$



Time of flight :

T = Time of ascent + Time of descent

$= t + t = 2t$

at H: $u_y = u\sin\theta$, $V_y = 0$, $a_y = -g$, $t = T/2$

$V_y = u_y + a_y t$

$0 = u\sin\theta + (-g)T/2$

$\Rightarrow \boxed{T = \frac{2u\sin\theta}{g}}$

Maximum Height :-

$u_y = u\sin\theta$, $a_y = -g$, $y_0 = 0$, $y = H$, $t = T/2 = \frac{u\sin\theta}{g}$

$y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$H = 0 + (u\sin\theta)\left(\frac{u\sin\theta}{g}\right) + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$

$H = \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\frac{u^2\sin^2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$

$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$

Horizontal Range, R

$R = (u\cos\theta) \times T$

$\Rightarrow R = (u\cos\theta) \times \left(\frac{2u\sin\theta}{g}\right)$

$\Rightarrow \boxed{R = \frac{u^2\sin 2\theta}{g}}$ | $\sin 2\theta = 2\sin\theta\cos\theta$

$R \rightarrow$ Maximum?

when $\sin 2\theta \rightarrow$ maximum

$\Rightarrow 2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

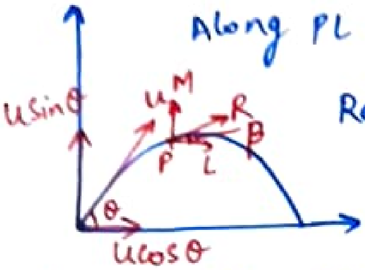
If angle of projection is $(90^\circ - \theta)$: $R' = \frac{u^2\sin 2(90^\circ - \theta)}{g}$

$= \frac{u^2\sin(180^\circ - 2\theta)}{g}$

$= \frac{u^2\sin 2\theta}{g}$ R

R: Same for $(45^\circ + \theta)$ & $(45^\circ - \theta)$

VELOCITY AT ANY TIME : $u_x = u\cos\theta$, $a_x = 0$, $u_y = u\sin\theta$, $a_y = -g$



Along PL : $V_x = u_x + a_x t = u\cos\theta$; Along PM: $V_y = u_y + a_y t = u\sin\theta - gt$

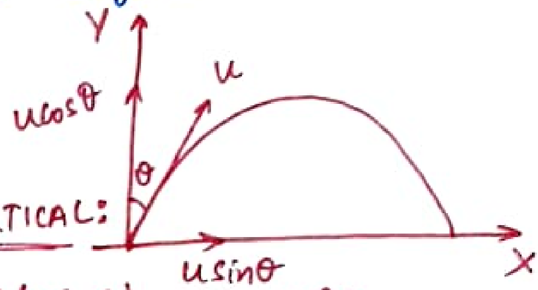
Resultant Velocity (PR) $= \sqrt{PL^2 + PM^2} = \sqrt{V_x^2 + V_y^2}$

$$V = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2(u\sin\theta)gt}$$

$$= \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

$$\tan\beta = \frac{RL}{PL} = \frac{PM}{PL} = \frac{V_y}{V_x}$$

$$\tan\beta = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$



PROJECTION AT AN ANGLE WITH THE VERTICAL:

$$T = \frac{2u\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}, \quad H = \frac{u^2\sin^2(90^\circ - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$R = \frac{u^2\sin 2(90^\circ - \theta)}{g} = \frac{u^2\sin 2\theta}{g}$$

$$y = x\tan(90^\circ - \theta) - \frac{1}{2}\frac{gx^2}{u^2\cos^2(90^\circ - \theta)} = x\cot\theta - \frac{1}{2}\frac{gx^2}{u^2\sin^2\theta}$$

$$v = \sqrt{u^2 + g^2t^2 - 2ugt\sin(90^\circ - \theta)} = \sqrt{u^2 + g^2t^2 - 2ugt\cos\theta}$$

$$\tan\beta = \tan(90^\circ - \theta) - \frac{gt}{u\cos(90^\circ - \theta)} = \cot\theta - \frac{gt}{u\sin\theta}$$

FLUID RESISTANCE:- With air resistance forces, the range & maximum height are smaller & shape is no longer symmetrical.

The projectile hits the ground at a Steeper angle