CLASS X : ASSIGNMENT : CHAPTER-1 : REAL NUMBERS : MATHEMATICS

1. Express each of the following positive integers as the product of its prime factors:
   (i) 234 (v) 468
   (ii) 420 (vi) 20570
   (iii) 945 (vii) 58500
   (iv) 7325 (viii) 45470971
2. Prove that the product of two consecutive positive integers is divisible by 2.
3. Prove that if x and y are odd positive integers, then $(x^2 + y^2)$ is even but not divisible by 4.
4. Show that one and only one out of n, n + 2, n + 4 is divisible by 3, where n is any positive integer.
5. Show that there is no natural number for which 4^n ends with the digit zero.
6. Find the HCF and LCM of the following pair of integers using the prime factorization method.
   (i) 26 and 91 (Ans: 182, 17) (iii) 84, 90 and 120 (Ans: 2520, 6)
   (ii) 40, 36 and 126 (Ans: 2520, 2) (iv) 24, 15 and 36 (Ans: 360, 3)
7. Can two numbers have 16 as their HCF and 380 as their LCM? Give reason. (Ans: No)
8. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725. Find the other. (Ans: 435)
9. The LCM and HCF of two numbers are 180 and 6 respectively. If one of the numbers is 30, find the other. (Ans: 36)
10. Find the smallest number which is divisible by both 306 and 657. (Ans: 22338)
11. Find the greatest number of six digits which is exactly divisible by 24, 15, and 36. (Ans: 999720)
12. Find the least number that is divisible by all the numbers between 1 and 10. (both inclusive) (Ans: 2520)
13. Find the largest number which divides 615 and 963 leaving the remainder 6 in each case. (Ans: 87)
14. Find the largest number that will divide 398, 436, and 542 leaving remainders 7, 11, and 15 respectively. (Ans: 17)
15. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468. (Ans: 4663)
16. A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles. (Ans: 4290)
17. A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60, and 72 km a day, round the field. when will they meet again? (Ans: 30 days)
18. Two tankers contain 850 l and 680 l of petrol respectively. Find the maximum capacity of a container that can measure the petrol of either tanker an exact number of times. (Ans: 170 l)
19. Prove that:
   (i) $\sqrt{2}$ is irrational (ii) $\sqrt{3}$ is irrational
   (iii) $\sqrt{5}$ is irrational
   (iv) $\sqrt{n}$ is irrational, where n is any prime positive integer.
20. Prove that the following are irrationals:
   (i) $\sqrt{2} + \sqrt{5}$ (iv) $2/\sqrt{7}$ (vii) $5\sqrt{2}$
   (ii) $3 - \sqrt{5}$ (v) $3/2\sqrt{5}$ (viii) $5 - 2\sqrt{3}$
   (iii) $\sqrt{5} + \sqrt{3}$ (vi) $4 + \sqrt{2}$ (ix) $(2 + \sqrt{3})/5$
21. Let a, b, c, d be positive rationals such that $a + \sqrt{b} = c + \sqrt{d}$, then either a = c and b = d or b and d are squares of rationals.
22. Let a, b, c, p be rational numbers such that p is not a perfect cube. If $a + bp^{1/3} + cp^{2/3} = 0$, then prove that a = b = c = 0.