PHYSICS INDUCTION
An institute of Science & Mathematics

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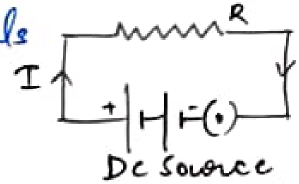
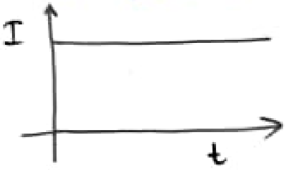
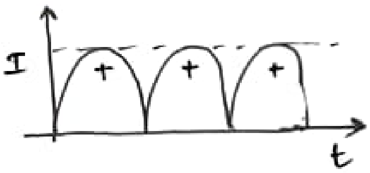
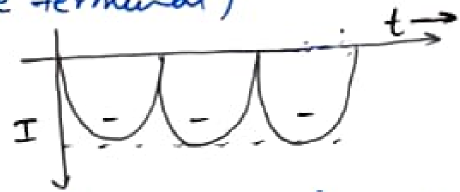
CLASS XII : NOTES : CHAPTER-7 : ALTERNATING CURRENT : PHYSICS

* When a resistor is connected across the terminals of a battery, a unidirectn current is established called direct Current (DC). The current has a unique directn (+ve terminal to −ve terminal)









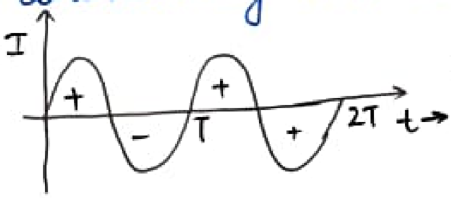
ALTERNATING CURRENT:- An Alternating current is the current whose magnitude changes continuously with time & whose direction is reversed periodically.

It's represented by:

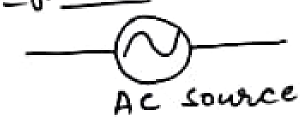
$$I = I_0 \sin \omega t$$

or

$$I = I_0 \cos \omega t$$

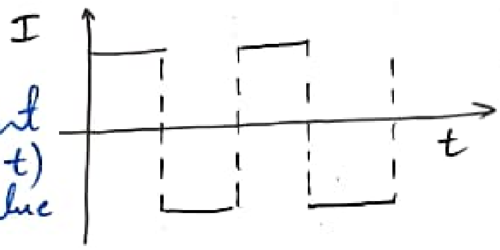


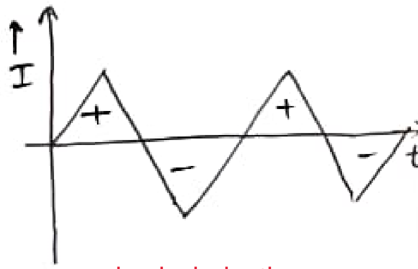
Symbol :-



Here, I: Instantaneous value of current (at time, t)

I_0: peak value or maxm value of a.c. (Amplitude)





ω: Angular frequency of a.c., $\omega = \frac{2\pi}{T} = 2\pi\nu$

T: Time period: time taken by a.c. to go thro' one complete cycle of variation

ν: Frequency: No. of complete cycles of variation by the a.c. in 1 sec.

Average Value of AC :- The average value of ac is defined as the average of all the instantaneous values of an alternating current over one complete cycle.

$$\boxed{y = f(t) \qquad \text{Average } y = \frac{\int y\,dt}{\int dt}}$$

Instantaneous current, $I = \frac{dq}{dt}$

$\Rightarrow \quad dq = I\,dt$

$\therefore \quad q = \int I\,dt$

Average Current, I_{avg} : Total charge flowing in a given time interval

$$I_{avg} = \frac{q}{t} = \frac{\int I\,dt}{\int dt}$$

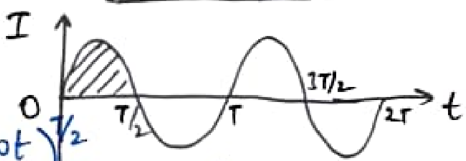
For full cycle: $(0 \to T)$

$$I_{avg} = \frac{\int_0^T I\,dt}{\int_0^T dt} = \frac{\int_0^T I_0 \sin\omega t\,dt}{\int_0^T dt} = \frac{I_0\left(\frac{-\cos\omega t}{\omega}\right)_0^T}{T}$$

$$\Rightarrow I_{avg} = \frac{-I_0(\cos\omega t)_0^T}{\omega T} = \frac{-I_0(\cos\omega T - \cos 0^\circ)}{\omega T} \qquad (\text{But } T = 2\pi/\omega)$$

$$\Rightarrow I_{avg} = \frac{-I_0(\cos 2\pi - \cos 0^\circ)}{2\pi} = \frac{-I_0(1-1)}{2\pi} = 0 \qquad \boxed{I_{avg} = 0}$$

For Half cycle: $(0 \to T/2)$



$$I_{avg} = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin\omega t\,dt}{\int_0^{T/2} dt} = \frac{I_0\left(\frac{-\cos\omega t}{\omega}\right)_0^{T/2}}{(T)_0^{T/2}} = \frac{-I_0\left(\cos\frac{\omega T}{2} - \cos 0^\circ\right)}{\omega T/2}$$

$$\Rightarrow I_{avg} = \frac{-I_0(\cos\pi - \cos 0^\circ)}{\pi} = \frac{2I_0}{\pi} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

* To get sinusoidally varying alternating current, we need a source (AC Generator) which can generate sinusoidally varying e.m.f.

AC Voltage, $V = V_0 \sin\omega t$

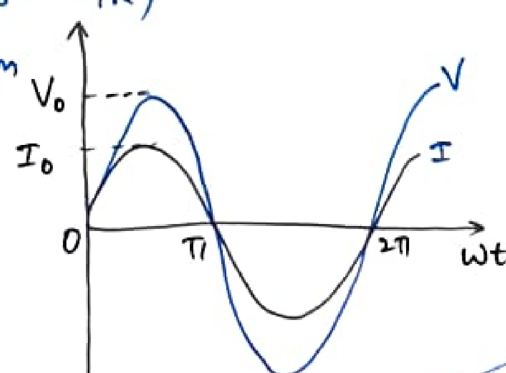
V_0 : Amplitude of oscillating p.d.

$V = IR$

$\Rightarrow V_0 \sin\omega t = IR$

$\Rightarrow I = \frac{V_0}{R}\sin\omega t = I_0 \sin\omega t \qquad (I_0 = V_0/R)$

* Both V & I reach zero, minimum or max^m values at the same time. clearly, V & I are in phase with each other



* for Half Cycle: $(0 - T/2)$

$V = V_0 \sin\omega t \qquad V_{avg} = \frac{2V_0}{\pi} = 0.637\,V_0$

$I = I_0 \sin\omega t \qquad I_{avg} = \frac{2I_0}{\pi} = 0.637\,I_0$

RMS (Root Mean Square) Value of AC

$I_{rms} = \sqrt{\langle I^2 \rangle} \qquad V_{rms} = \sqrt{\langle V^2 \rangle}$

1st → Square
2nd → Average
3rd → Root

Voltage (Home) - 220 V 50 Hz
↑ rms value

As, $I = I_0 \sin\omega t$

$$\Rightarrow I_{rms} = \sqrt{\langle I^2 \rangle} = \left(\langle I_0^2 \sin^2\omega t \rangle\right)^{1/2} = \left(\frac{\int_0^T I_0^2 \sin^2\omega t \, dt}{\int_0^T dt}\right)^{1/2}$$

$$\therefore I_{rms} = \left(\frac{I_0^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt\right)^{1/2}$$

$\left[\text{As } 2\sin^2\theta = 1-\cos 2\theta \Rightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$

$$\Rightarrow I_{rms} = \left[\frac{I_0^2}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T\right]^{1/2}$$

$$\Rightarrow I_{rms} = \left[\frac{I_0^2}{2T}\left(T - \frac{\sin 2\omega T}{2\omega} - 0 + \frac{\sin 0^\circ}{2\omega}\right)\right]^{1/2}$$

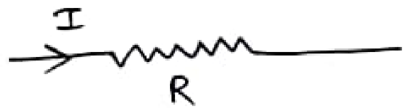
$\left(\because T = \frac{2\pi}{\omega}, \sin 4\pi = 0\right)$

$$= \left[\frac{I_0^2}{2T}(T - 0 - 0 + 0)\right]^{1/2} = \left(\frac{I_0^2}{2}\right)^{1/2} \Rightarrow \boxed{I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0}$$

• rms value is also called effective value of a.c. or virtual value of a.c.

for alternating voltage, $V = V_0 \sin\omega t$: $\boxed{V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707 V_0}$

Energy dissipated in Resistor in time t



$E = I^2 R t$, $I = I_0 \sin\omega t$

Energy dissipated in small time, dt : $\int dE = \int I^2 R \, dt$

$$\therefore E = R\int I^2 dt = R (I_{rms})^2 \times t$$

$$\because I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\int I^2 dt}{\int dt}}$$

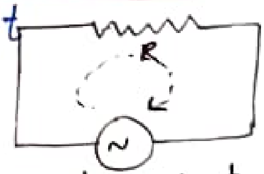
Significance of RMS

$$\Rightarrow I_{rms}^2 = \frac{\int I^2 dt}{\int dt} \Rightarrow \int I^2 dt = (I_{rms})^2 \times t$$

Note: DC devices can't measure AC and EMF. They will show zero reading for AC. So, Hot wire Ammeter & Hot wire Voltmeter are used to measure AC (RMS) & voltage (RMS).

CIRCUIT THEORY

AC CIRCUIT CONTAINING RESISTANCE ONLY (Pure Resistive Circuit):-



$V = V_0 \sin\omega t$

Let I be the current in the circuit at any instant, t

The p.d. developed across R will be IR.

$V - IR = 0$ (Kirchhoff's Law)

$\Rightarrow V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{V_0 \sin\omega t}{R} = I_0 \sin\omega t \quad \left(\because I_0 = \frac{V_0}{R}\right)$$

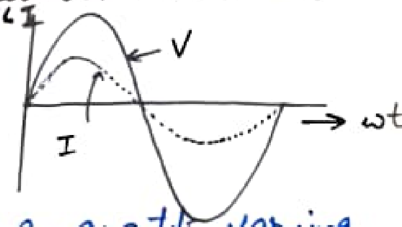
Peak Potential

Peak Current

* The behaviour of R in d.c. & a.c. circuits is the same. R can reduce a.c. equally effectively as it can reduce d.c.

As, $I = I_0 \sin\omega t$ & $V = V_0 \sin\omega t$ ∴ phase difference $= \omega t - \omega t = 0$

* In an ac circuit containing R only, the voltage & current are in the same phase. ⇒ minima, maxima & zero of alternating voltage & alternating current in a pure resistor occur at the same respective times.

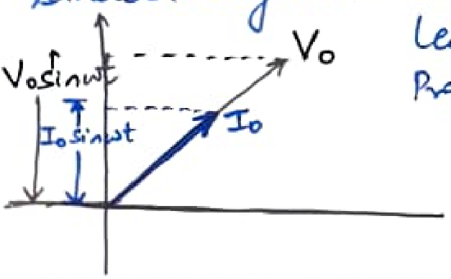


* **Phasor Diagram** :- A rotating vector that represents a quantity varying sinusoidally with time is called a phasor.

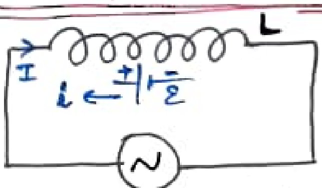
Length of arrow → Peak Value (V_0, I_0)

Projection on Y-axis → Instantaneous value

$$\boxed{I_0 = \frac{V_0}{R} \quad \therefore I_0 < V_0}$$



AC CIRCUIT CONTAINING INDUCTANCE ONLY (Pure Inductive Circuit):



If $\frac{dI}{dt}$ is the rate of change of current thro' L at any instant, then induced emf in the inductor at the same instant is $\varepsilon = -L\frac{dI}{dt}$.

$V = V_0 \sin\omega t$

(−ve sign : induced emf opposes the change of current)

Assuming that Resistance of the inductor is negligible. The circuit is therefore, a purely inductive circuit.

To maintain the flow of current, applied voltage V must be equal & opposite to the induced voltage, ε

$V = -\varepsilon$

$\Rightarrow V - L\frac{dI}{dt} = 0$

Or, Using Kirchhoff's loop rule, $\Sigma \varepsilon(t) = 0$ ∵ R = 0

$\Rightarrow V - \varepsilon = 0$ or $V - L\frac{dI}{dt} = 0$

$$\therefore \frac{dI}{dt} = \frac{V}{L} = \frac{V_0 \sin\omega t}{L}$$

$$\Rightarrow dI = \frac{V_0}{L} \sin\omega t \, dt$$

On integrating, $\int dI = \frac{V_0}{L} \int \sin\omega t \, dt$

$$\Rightarrow I = \frac{V_0}{L}\left(\frac{-\cos\omega t}{\omega}\right) + \text{const} = \frac{-V_0 \cos\omega t}{\omega L} + \text{constant}$$

{ Integration const is time indep., ∴ constant or time-indep comp of the current exists ∴ Const = 0

$$I = \frac{-V_0 \cos\omega t}{\omega L} = -I_0 \cos\omega t \quad \left(\text{where, } I_0 = \frac{V_0}{\omega L}\right)$$

I_0 : Amplitude of the current & the quantity ωL is analogous to the Resistance & is called inductive reactance, denoted by X_L : $X_L = \omega L$

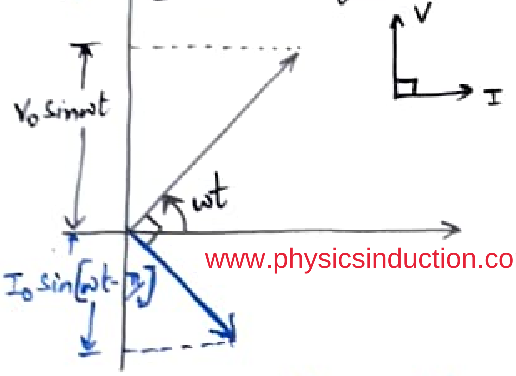
dim.$[X_L]$ = dim $[R]$ =   S.I. Unit of $X_L = \Omega$

As $I = -I_0 \cos\omega t = -I_0 \sin\left(\frac{\pi}{2} - \omega t\right) = \underline{I_0 \sin\left(\omega t - \pi/2\right)}$ ← Current lags behind potential by $90°$ i.e. by $1/4$th of a period

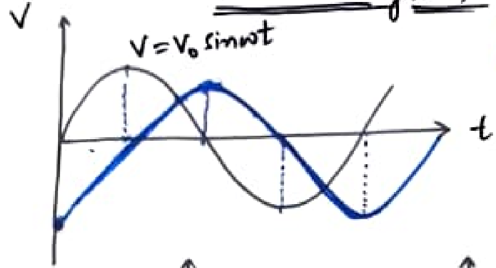
And $\underline{V = V_0 \sin\omega t}$ ⇒ The current phasor I is $\pi/2$ behind the voltage phasor, V.

i.e.,

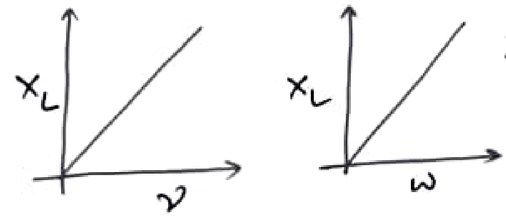
Phasor Diagram:



Wave Diagram:-



Current is max when pot is min & min when pot. is maximum.



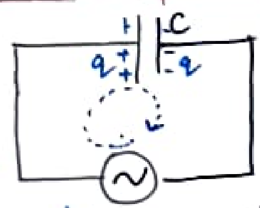
$X_L \propto \omega$
$X_L \propto \nu$
$X_L \propto L$

X_L Vs ν :- $X_L = \omega L = (2\pi\nu)L$

In d.c. circuits: $\nu = 0$ ∴ $X_L = 0$

A pure inductance offers zero resistance to dc. It means a pure inductor can't reduce dc.

AC CIRCUIT CONTAINING CAPACITANCE ONLY: Pure Capacitive Circuit:-



The current flowing in the circuit transfers charge to the plates of the capacitor. This produces a p.d. b/w the plates. The capacitor is alternately charged & discharged as the current reverses each half cycle. $\underline{V = V_0 \sin\omega t}$ At any instant, t suppose q is the charge on the capacitor. Therefore, p.d. across the plates of capacitor, $V = q/C$

From the Kirchhoff's loop Rule : $V - \frac{q}{C} = 0 \Rightarrow V = \frac{q}{C} \Rightarrow q = CV$

$\Rightarrow q = CV_0 \sin\omega t$

$\Rightarrow I = \frac{dq}{dt} = CV_0(\cos\omega t).\omega = I_0 \cos\omega t$, where $I_0 = \omega C V_0 = \frac{V_0}{1/\omega C} = \frac{V_0}{X_C}$

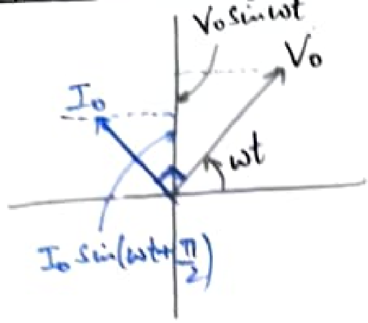
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$ (X_C → Capacitive Reactance)

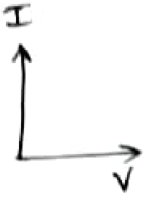
As, $\underline{I} = I_0 \cos\omega t = \underline{I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$

$\cos\omega t = \sin\left(\frac{\pi}{2} - \omega t\right)$ can't be used ∵ we have to compare I with $V = V_0 \sin\omega t$ which involves $+\omega t$ & not $(-\omega t)$.

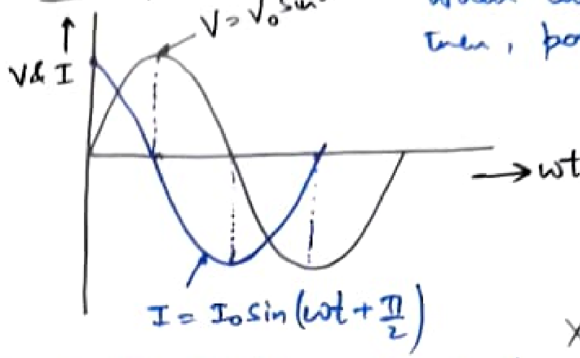
In an AC circuit containing C only, current leads pot, V by a phase angle of $90°$.

Phasor Diagram:

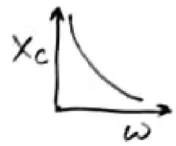


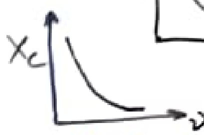


Waveform :-



when current is max^m then, pot. is minimum.

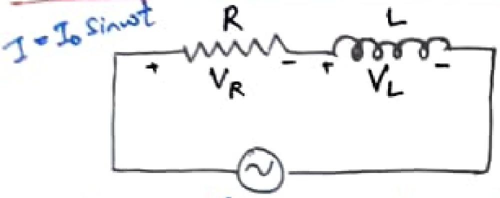




X_C Vs ν :- $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$; $X_C \propto \frac{1}{\nu}$, $X_C \propto \frac{1}{C}$

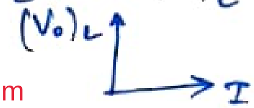
In a d.c. circuit: $\nu = 0$ $\therefore$ $X_C = \infty$ i.e., a condenser will block d.c.

SERIES L-R CIRCUIT : AC Circuit containing Resistance & Inductance :-

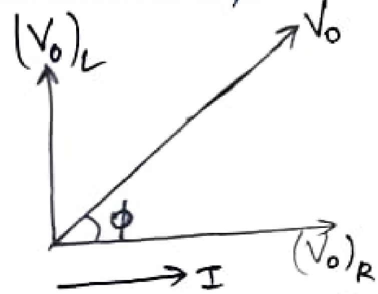


Let a source of alternating emf be connected to an ohmic Resistance, R & a coil of inductance, L in series.

$V_R(t) = (V_0)_R \sin\omega t$ Illy, $V_L(t) = (V_0)_L \sin(\omega t + \pi/2)$



$\therefore$



$\therefore V_0^2 = (V_0)_R^2 + (V_0)_L^2$

$V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$

$= \sqrt{I_0^2 R^2 + I_0^2 X_L^2}$

$= I_0 \sqrt{R^2 + X_L^2}$

$= I_0 Z$

where, Z : Impedence

$Z = \sqrt{R^2 + X_L^2}$

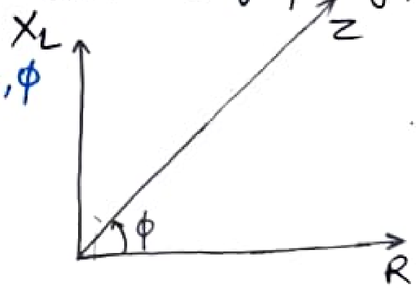
we find that in L-R circuit, Current lags pot by ϕ

Voltage leads the Current by phase angle, ϕ

where

$\tan\phi = \frac{V_L}{V_R}$

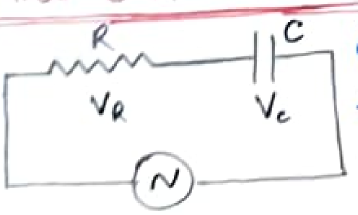
$= \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R}$



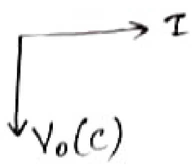
$\because (V_0)_R = I_0 R$

$(V_0)_L = I_0 X_L$

SERIES C-R CIRCUIT : AC CIRCUIT CONTAINING RESISTANCE & CAPACITANCE



Consider an alternating source of emf connected to an ohmic Resistance, R & a condenser of capacity, C in series.



$I = I_0 \sin\omega t$

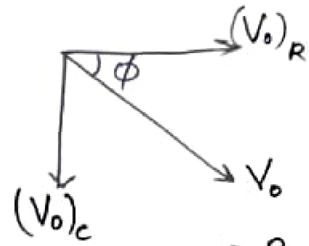
$V_R(t) = (V_0)_R \sin\omega t$ (same phase)

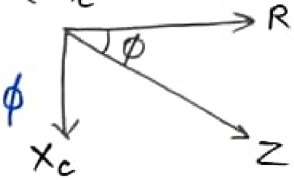
$V_C(t) = (V_0)_C \sin(\omega t - \pi/2)$ (V lags I by 90°)

$$V_0^2 = (V_0)_R^2 + (V_0)_C^2$$

$$\Rightarrow V_0 = \sqrt{(V_0)_R^2 + (V_0)_C^2} = \sqrt{I_0^2R^2 + I_0^2X_C^2} = I_0\sqrt{R^2+X_C^2} = I_0 Z$$

where, $Z = \sqrt{R^2+X_C^2}$

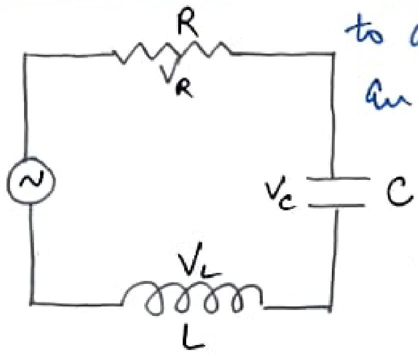




from the phasor diagram, we find that in CR circuit, voltage lags behind the current by a phase angle, ϕ

$$\tan\phi = \frac{(V_0)_C}{(V_0)_R} = \frac{I_0X_C}{I_0R} = \frac{X_C}{R} \qquad \left[\because (V_0)_R = I_0R,\ (V_0)_C = I_0X_C\right]$$

SERIES LCR CIRCUIT: AC Circuit containing Resistance, Inductance & Capacitance :-



Consider an alternating source of emf connected to an ohmic Resistance, R, Condenser, C & and an inductor, L.

$$I = I_0 \sin\omega t$$

$$V_R(t) = (V_0)_R \sin\omega t \quad ; \quad (V_0)_R = I_0R$$

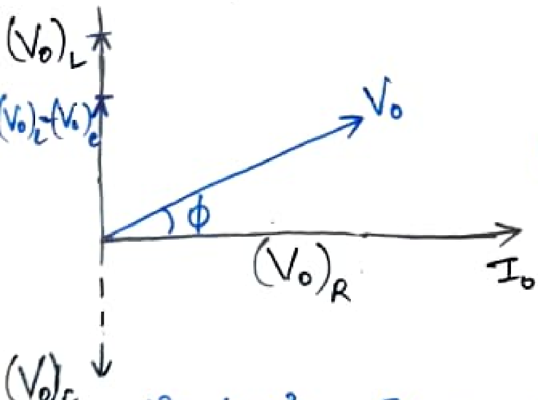
$$V_L(t) = (V_0)_L \sin\left(\omega t + \frac{\pi}{2}\right) \ ; \ (V_0)_L = I_0X_L,\ X_L = \omega L$$

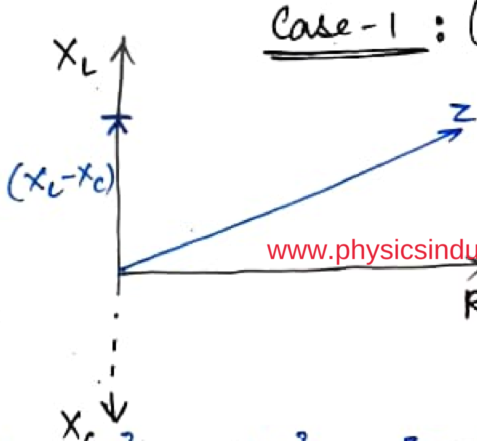
$$V_C(t) = (V_0)_C \sin\left(\omega t - \frac{\pi}{2}\right) \ ; \ (V_0)_C = I_0X_C,\ X_C = \frac{1}{\omega C}$$

Phasor diagram :-

Case-1 : $(V_0)_L > (V_0)_C$ OR $X_L > X_C$

$$V = V_0 \sin(\omega t + \phi)$$





$$V_0^2 = (V_0)_R^2 + \left[(V_0)_L - (V_0)_C\right]^2 = I_0^2R^2 + \left[I_0X_L - I_0X_C\right]^2$$

$$\Rightarrow V_0^2 = I_0^2\left[R^2 + (X_L - X_C)^2\right]$$

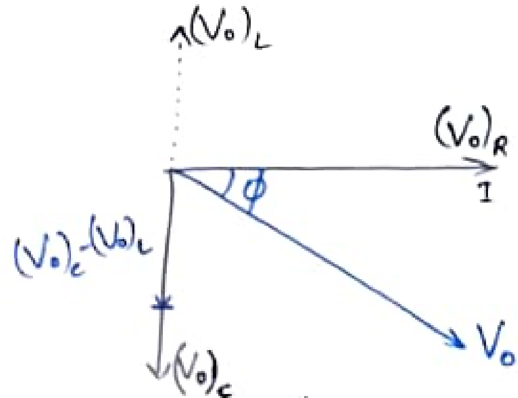
$$\Rightarrow V_0 = I_0\sqrt{R^2 + (X_L-X_C)^2}$$

$$\Rightarrow V_0 = I_0 Z \qquad \text{where } Z(\text{Impedance}) = \sqrt{R^2 + (X_L - X_C)^2}$$

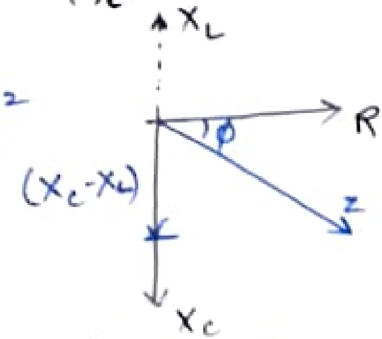
From the phasor diagram, it's clear that in LCR circuit, Voltage leads the current by a phase angle, ϕ. $\Rightarrow$ Inductance dominated circuit (lead Volt. & $X_L > X_C$)

$$\tan\phi = \frac{(V_0)_L - (V_0)_C}{(V_0)_R} = \frac{I_0X_L - I_0X_C}{I_0R} = \frac{X_L - X_C}{R}$$

Case-2: $(V_0)_c > (V_0)_L \Rightarrow X_c > X_L$



$$V_0^2 = (V_0)_R^2 + [(V_0)_c - (V_0)_L]^2$$
$$= I_0^2R^2 + [I_0X_c - I_0X_L]^2$$
$$= I_0^2[R^2 + (X_c - X_L)^2]$$
$$\Rightarrow V_0 = I_0\sqrt{R^2 + (X_c - X_L)^2}$$
$$= I_0 Z \quad \text{where, } Z = \sqrt{R^2 + (X_c - X_L)^2}$$



From the phasor diagram, it's clear that voltage lags behind the current by a phase angle, ϕ.

$V = V_0 \sin(\omega t - \phi)$: Capacitance dominated circuit.

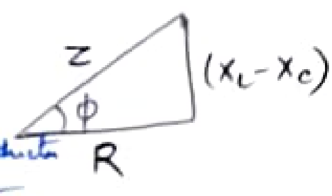
$$\tan\phi = \frac{(V_0)_c - (V_0)_L}{(V_0)_R} = \frac{I_0X_c - I_0X_L}{I_0 R} = \frac{X_c - X_L}{R}$$

Case-3: $(V_0)_L = (V_0)_c \Rightarrow X_L = X_c$

$\therefore V_0 = (V_0)_R$: Pure Resistive circuit

V_0 & I_0 are in the same phase, $\therefore \phi = 0°$ & $Z = R$ $\left(\begin{matrix} V_0 = I_0 Z \\ = I_0 R \end{matrix}\right)$

Impedance Triangle:-



R : Ohmic Resistance (arises on account of mat. of the cond, $R = \rho l/A$)

X_L: Inductive Reactance, $X_L = \omega L$: Resist offered by Inductor

X_c: Capacitive " , $X_c = 1/\omega C$: Resist offered by capacitor

Total Reactance $= \pm(X_L - X_c)$

V leads I   V lags I

Z: Impedance : Net effective Resist offered by LCR circuit

$$Z = \sqrt{R^2 + (X_L - X_c)^2}, \quad \tan\phi = \frac{X_L - X_c}{R}$$

Susceptance:- The Reciprocal of Reactance is called susceptance of a.c. circuit

Admittance:- The Reciprocal of impedance is called Admittance of a.c. circuit

POWER IN AC CIRCUITS : THE POWER FACTOR

In a d.c. circuit : $P = VI$

In an a.c. circuit : Values of voltage & current change every instant

$\therefore$ Power in an a.c. circuit at any instant is the product of instant. voltage, $V(t)$ & instantaneous current, $I(t)$ $\therefore$ Power will also vary with time, $P(t)$

$\therefore$ Instantaneous Power, $P(t) = V(t)\,I(t)$

Let $V(t) = V_0 \sin\omega t$ & $I = I_0 \sin(\omega t + \phi)$

$$\therefore \quad P(t) = V_0 \sin\omega t \; I_0 \sin(\omega t + \phi)$$
$$= V_0 I_0 \sin\omega t \sin(\omega t + \phi)$$

AVERAGE POWER : $P_{avg} = \langle P(t) \rangle$ or $\overline{P(t)}$

As, $P(t) = V_0 I_0 \sin\omega t \sin(\omega t + \phi)$

$$= V_0 I_0 \left(\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi\right)$$
$$= V_0 I_0 \left(\sin^2\omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi\right)$$

$\therefore \langle P(t) \rangle$ (over a complete cycle) $= \dfrac{\int_0^T P.dt}{\int_0^T dt} = \dfrac{\int_0^T (V_0 I_0 \sin^2\omega t \cos\phi)\,dt + \int_0^T \left(\frac{\sin 2\omega t}{2} \sin\phi\right)dt}{T}$

$$\Rightarrow \langle P(t) \rangle = \frac{\frac{V_0 I_0 \cos\phi}{2} \int_0^T (1 - \cos 2\omega t)\,dt + \frac{\sin\phi}{2} \int_0^T \sin 2\omega t \, dt}{T}$$

$$= \frac{V_0 I_0 \cos\phi}{2T} \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T + \frac{\sin\phi}{2T} \left[-\frac{\cos 2\omega t}{2\omega}\right]_0^T$$

$(\omega = 2\pi/T)$

$$= \frac{V_0 I_0 \cos\phi}{2T} \left[T - \frac{\sin 4\pi}{2\omega} + 0 - 0\right] - \frac{\sin\phi}{2T} \left[\frac{\cos 4\pi}{2\omega} - \frac{\cos 0^\circ}{2\omega}\right]$$

$$= \frac{V_0 I_0 \cos\phi}{2} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cdot \cos\phi = V_{rms} I_{rms} \cos\phi$$

$$\therefore \boxed{\text{Average Power} = V_{rms} I_{rms} \cos\phi = I_{rms}^2 Z \cos\phi = \frac{V_{rms}^2}{Z} \cos\phi}$$

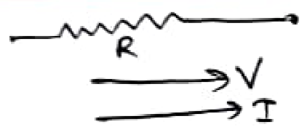
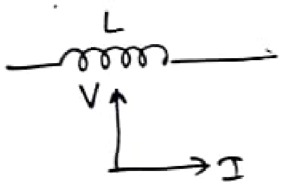
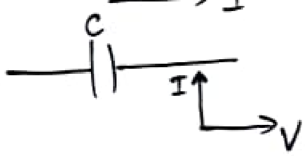
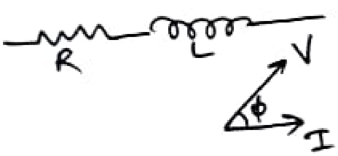
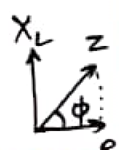
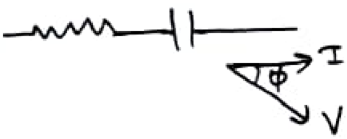
$\therefore$ Avg. Power dissipated depends on V, I & cosine of phase angle

Here, $\cos\phi$ - Power factor

POWER FACTOR :- Power factor of an a.c. circuit is defined as the ratio of true power to apparent power of the circuit.

P: True Power, $V_{rms} I_{rms}$: Apparent Power

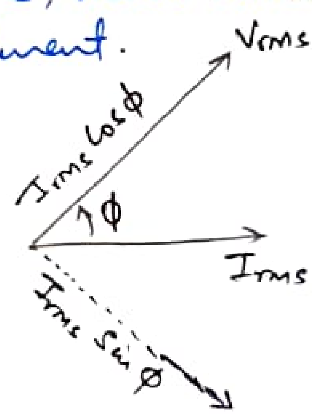
$$\text{Power factor} = \cos\phi = \frac{P}{V_{rms} I_{rms}}$$

CIRCUITS	Phase difference	Power factor, $\cos\phi$	Avg. Power, $\langle P(t) \rangle$
1) 	$0°$	1	$V_{rms} I_{rms}$
2) 	$90°$	0	zero
3) 	$90°$	0	zero
4) 	ϕ	$\frac{R}{Z}$  $Z = \sqrt{X_L^2 + R^2}$	$V_{rms} I_{rms} \cos\phi$
5) 	ϕ	$\frac{R}{Z}$  $Z = \sqrt{R^2 + X_C^2}$	$V_{rms} I_{rms} \cos\phi$

$$\text{Also, } \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$\cos\phi \leq 1$ & It's always positive

Wattless Current / Idle Current :- The current which consumes no power for its maintenance in the circuit is called wattless current or Idle current. Current thro' pure L or pure C, which consumes no power for its maintenance is called Idle current.



Phase angle b/w V_{rms} & $I_{rms}\cos\phi = 0°$

$\therefore P_{avg}(\text{due to } I_{rms}\cos\phi) = V_{rms}(I_{rms}\cos\phi)\cos 0° = V_{rms} I_{rms}\cos\phi$

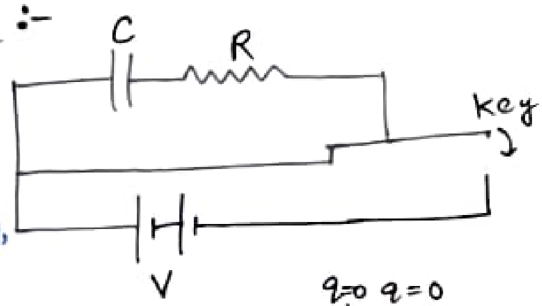
& Phase angle b/w V_{rms} & $I_{rms}\sin\phi = 90°$

$\therefore P_{avg}(\text{due to } I_{rms}\sin\phi) = V_{rms}(I_{rms}\sin\phi)\cos 90° = 0$

$I_{rms}\sin\phi$ gives no contribution to power consumption in AC $\therefore$ Idle Current

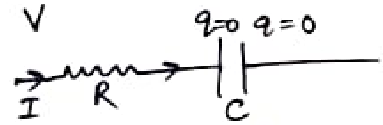
CHARGING AND DISCHARGING OF CAPACITOR: GROWTH AND DECAY OF CHARGE IN A CAPACITOR: R-C CIRCUIT

Charging of a capacitor through a Resistor :-



Let a battery of emf, V be connected to a capacitor of capacitance, C & a resistor of resistance, R in series thro' a morse key, K.

Initially (at $t=0$), capacitor is uncharged.

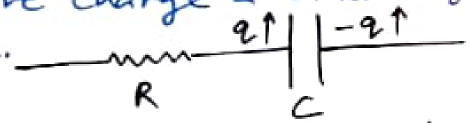


$q=0$ on capacitor.

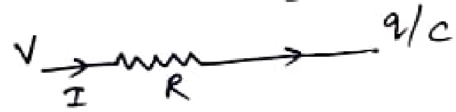
$\therefore$ p.d. across capacitor $= \frac{q}{C} = 0$ $(\because q=0)$ but $I = \frac{V}{R}$ ($\exists$ I in R)

Max^m current

With time, one plate of capacitor acquires +ve charge & other acquires equal -ve charge. $\therefore$ p.d. across C will inc.



$\therefore$ p.d. across capacitor $= \frac{q}{C}$ (at time, t)



& $I = \dfrac{V - \frac{q}{C}}{R}$

$\boxed{q\uparrow \Leftrightarrow I\downarrow \text{ (Charging)}}$

$$\Rightarrow \frac{dq}{dt} = \frac{CV-q}{RC} \quad (\because I = dq/dt)$$

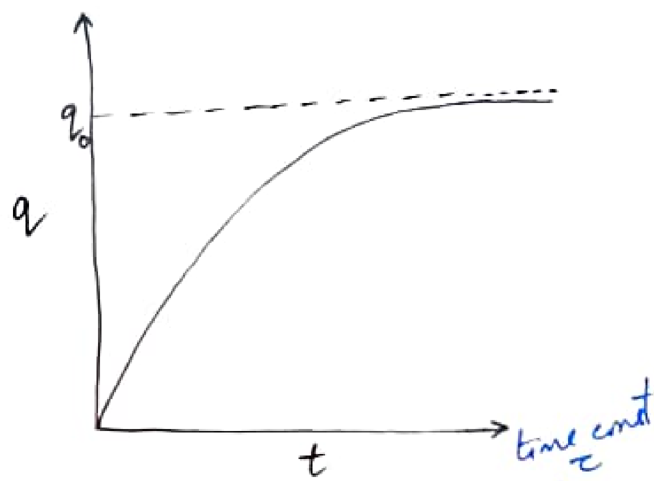
$$\Rightarrow \int_0^q \frac{dq}{CV-q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \left|\frac{\ln(CV-q)}{(-1)}\right|_0^q = \frac{1}{RC}(t)_0^t$$

$$\Rightarrow \ln(CV-q) - \ln(CV-0) = \frac{-t}{RC}$$

$$\Rightarrow \ln\left(\frac{CV-q}{CV}\right) = \frac{-t}{RC}$$

$$\Rightarrow \frac{CV-q}{CV} = e^{-t/RC}$$

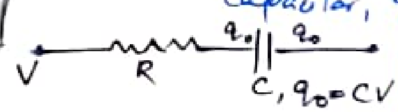


$$\Rightarrow 1 - \frac{q}{CV} = e^{-t/RC} \Rightarrow \frac{q}{CV} = 1 - e^{-t/RC} \Rightarrow q = CV(1-e^{-t/RC})$$

Max^m charge on capacitor, q_0

At $t=\infty$, $\boxed{q_0 = CV}$ $\quad \Rightarrow \boxed{q = q_0(1-e^{-t/RC})}$

where time const $= \tau = RC$



At $t = \tau = RC$

$$q = q_0\left(1 - e^{-RC/RC}\right) = q_0(1 - e^{-1}) = q_0\left(1 - \frac{1}{e}\right) = q_0\left(1 - \frac{1}{2.718}\right)$$

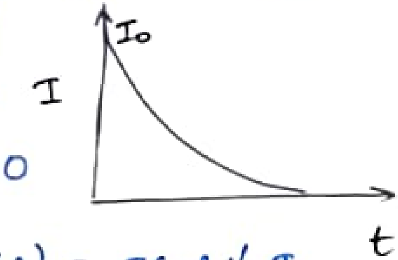
$$\Rightarrow q = q_0(1 - 0.368) = 0.632 q_0 = 63.2\% \, q_0$$

We may define <u>time constant</u> of RC circuit as the time in which charge on the capacitor grows to 63.2% of max^m value of charge.

I vs t

As, Current $I = \frac{dq}{dt} = \frac{d}{dt}\left[q_0\left(1 - e^{-t/RC}\right)\right] = \frac{q_0}{RC} e^{-t/RC} = \frac{CV}{RC} \cdot e^{-t/RC}$

$$\Rightarrow \underline{I = \frac{V}{R} e^{-t/RC}} = \boxed{I_0 e^{-t/RC} = I_0 e^{-t/\tau}}$$

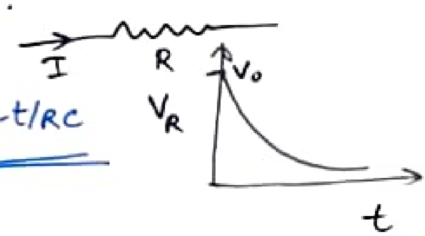


At $t = 0$, $I = \frac{V}{R} = \text{max}$, At $t = \infty$, $I = 0$

At $t = \tau$, $I = I_0 e^{-\tau/\tau} = I_0\left(\frac{1}{e}\right) = I_0(0.368) = 36.8\% \, I$

we may define, time const of RC circuit as the time in which current reduces to 36.8% of its initial value.

pot drop in R & C vs t

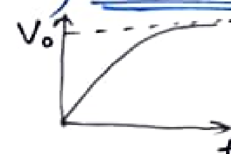


$$\underline{V_R} = IR = \left(I_0 e^{-t/RC}\right) \cdot R$$

$$= \frac{V_0}{\cancel{R}} e^{-t/RC} \cdot \cancel{R} = \underline{V_0 e^{-t/RC}}$$

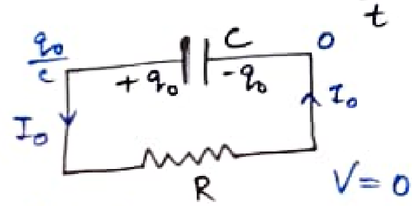
At $t = \tau = RC = V_R = V_0(0.368) = 36.8\% \, V_0$

& $\underline{V_C} = \frac{q}{C} = \frac{q_0\left(1 - e^{-t/RC}\right)}{C} = \frac{CV_0}{C}\left(1 - e^{-t/RC}\right) = V_0\left(1 - e^{-t/RC}\right) = \underline{V_0\left(1 - e^{-t/\tau}\right)}$



At $t = \infty$, $\underline{V_C} = V_0(1 - 0) = \underline{V_0}$

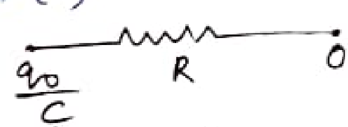
discharging of a capacitor through Resistor :-



at $t = 0$, capacitor is fully charged (q_0)

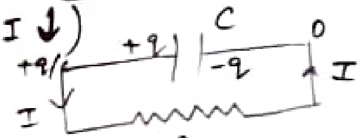
Pot. across capacitor $= \frac{q_0}{C}$

I flows from a higher pot $\left(\frac{q_0}{c}\right)$ to lower potential (0)

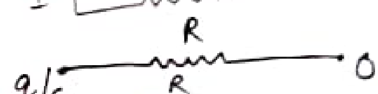


Initially, Current, $I_0 = \frac{\frac{q_0}{C} - 0}{R} = \frac{q_0}{RC}$ (max current)

$(q\downarrow \Leftrightarrow I\downarrow)$



After time, t; Current, $I = \frac{\frac{q}{C} - 0}{R} = \frac{q}{RC}$



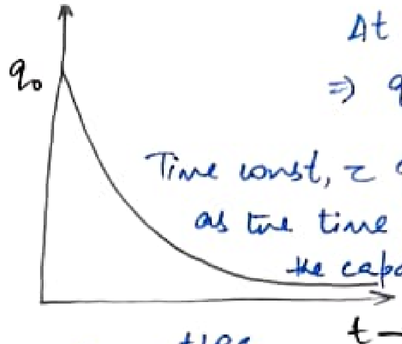
$$\Rightarrow -\frac{dq}{dt} = \frac{q}{RC} \Rightarrow \int_{q_0}^{q} \frac{dq}{q} = \int_{0}^{t} \frac{-dt}{RC} \Rightarrow \ln q \Big|_{q_0}^{q} = -\frac{1}{RC} \times t \Big|_0^t$$

(-ve sign : decay of q)

$$\ln q - \ln q_0 = \frac{-t}{RC}$$

$$\Rightarrow \ln\left(\frac{q}{q_0}\right) = \frac{-t}{RC}$$

$$\Rightarrow \boxed{q = q_0 e^{-t/RC} = q_0 e^{-t/\tau}}$$

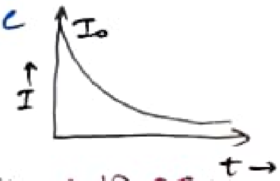


At $t = \tau$

$\Rightarrow q = q_0\left(\frac{1}{e}\right) = 0.368\, q_0$
$= 36.8\%\, q_0$

Time const, τ of RC circuit is defined as the time in which charge on the capacitor dec. to 36.8% of the max value

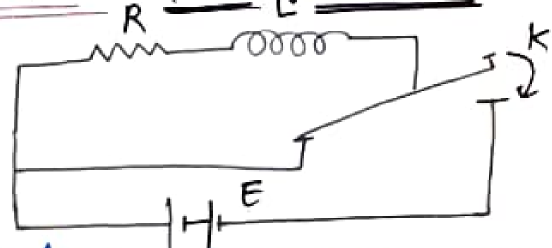
As, $I = \frac{-dq}{dt} = -q_0 e^{-t/RC}\left(-\frac{1}{RC}\right) = \frac{q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$



At $t = \tau$ $\quad I = I_0\left(\frac{1}{e}\right) = 0.368\, I_0 = 36.8\%\, I_0$

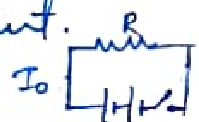
CHARGING AND DISCHARGING OF INDUCTOR: GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT: L-R CIRCUIT

Growth of Current



Consider a resistor of Resistance, R & a coil of inductance L connected to a battery. E thro' a morse key, K.

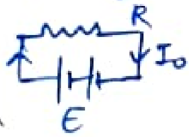
On pressing K, the battery is connected. Current grows in the L-R circuit. Due to self-Induction, an induced emf is set up across L. By Lenz's law, Induced emf opposes the growth of current.



At $t = 0$, $I = 0$

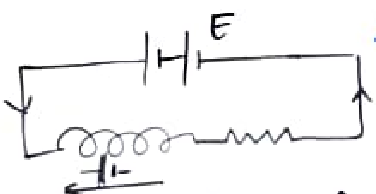
I reaches its max^m value, when there is no inductor $\therefore I_{max} = I_0 = \frac{E}{R}$

At $t = \infty$, $I = I_0 =$ const (steady state): Inductor plays no role, $I_0 = \frac{E}{R}$



At time t $(0 < t < \infty)$, $0 < I < I_0$

I is the strength of current at any instant, t & $\frac{dI}{dt}$ is the rate of growth of current at that instant.



Induced Current

$$|e| = L\frac{dI}{dt}$$

Pot. diff across $R = IR$

" " " $L = \frac{LdI}{dt}$

$$E - \frac{LdI}{dt} - IR = 0 \text{ (Kirchhoff's Rule : } \Sigma \Delta V = 0)$$

$$\Rightarrow \int_0^t \frac{dt}{L} = \int_0^I \frac{dI}{E - IR}$$

$$\Rightarrow \frac{t}{L} = \left.\frac{\ln|E - IR|}{-R}\right|_0^I \Rightarrow \ln(E - IR)\Big|_0^I = -\frac{Rt}{L} \Rightarrow \ln|E - IR| - \ln E = \frac{-Rt}{L}$$

$$-\frac{Rt}{L} = \ln\left|\frac{E-IR}{E}\right| \Rightarrow -\frac{R}{L}t = \ln\left|1-\frac{IR}{E}\right| \Rightarrow 1-\frac{IR}{E} = e^{-(R/L)t}$$

$$\Rightarrow \frac{IR}{E} = 1-e^{-(R/L)t} \Rightarrow I = \frac{E}{R}\left[1-e^{-(R/L)t}\right] \Rightarrow I = I_0\left[1-e^{-\left(\frac{R}{L}\right)t}\right]$$

$$\therefore \boxed{I = I_0\left[1-e^{-\left(\frac{R}{L}\right)t}\right] = I_0\left[1-e^{-t/\tau}\right]} \quad \text{where } \tau = \frac{L}{R}$$

unit of time

Inductive time const

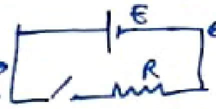
At $t=\tau$, $I = I_0\left(1-e^{-1}\right) = I_0\left(1-\frac{1}{e}\right)$

$= I_0(1-0.368) = 0.632 I_0 = 63.2\% I_0$

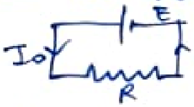
We may define time const. of LR circuit as the time in which current in the circuit grows to 63.2% of the max^m value of current.

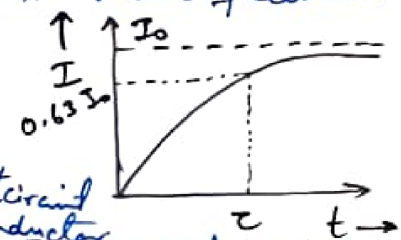
As $I = I_0\left[1-e^{-t/\tau}\right]$

At $t=0$, $I=0$

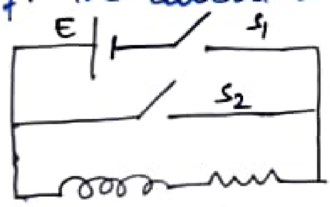


At $t=\infty$ (steady state), $I = I_0$





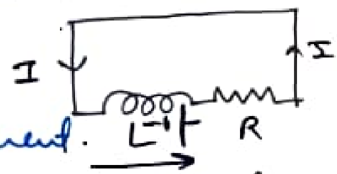
Decay of Current:- When the battery is cut off. The current in the LR circuit decays.



As, current has reached its max value, $I_0 = \frac{E}{R}$

open S_1 & close S_2

at time t, let I be the current flowing in the circuit. Current doesn't decay instantly b'coz induced current by L will support the decay of current & oppose the change in current.



$e = \frac{L\,dI}{dt}$ (+ve ∵ gain in pot.)

At $t=0$, $I = I_0$
At $t=\infty$, $I = 0$

$$-\frac{L\,dI}{dt} - IR = 0$$

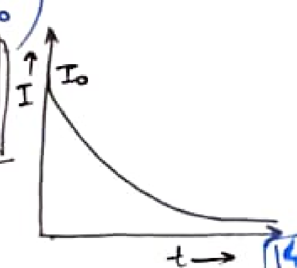
∵ $\frac{dI}{dt}$ is $-ve$ $\Rightarrow \frac{L\,dI}{dt} = -IR$

$$\therefore \int_0^t -\frac{R}{L}\,dt = \int_{I_0}^{I} \frac{dI}{I}$$

Inductive time const of LR Circuit is defined as the time in which current decays to 36.8% of the max^m value

$$\Rightarrow -\frac{R}{L}t = \left|\ln I\right|_{I_0}^{I} \Rightarrow -\frac{R}{L}t = \ln I - \ln I_0 = \ln\left(\frac{I}{I_0}\right)$$

$$\Rightarrow \frac{I}{I_0} = e^{-(R/L)t} \Rightarrow \boxed{I = I_0 e^{-(R/L)t} = I_0 e^{-t/\tau}}$$



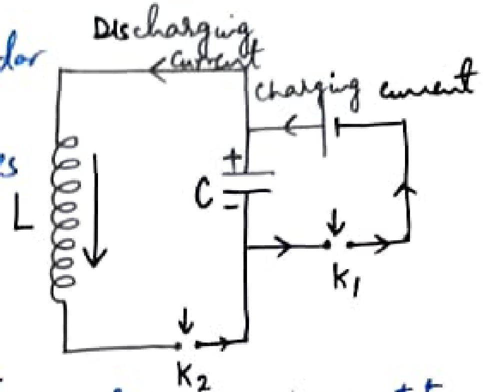
At $t=\tau$, $I = I_0\left(\frac{1}{e}\right) = 0.368 I_0 = 36.8\% I_0$

LC OSCILLATIONS :- A capacitor & an inductor can store electrical & magnetic energy resp.

When plug of K_1 is put in, the cell charges the capacitor to a pot., $V = q/C$

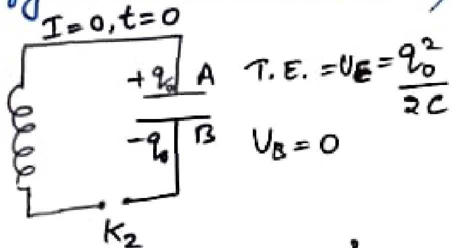
$q \rightarrow$ charge on capacitor plates

$V \rightarrow$ Voltage across the plates

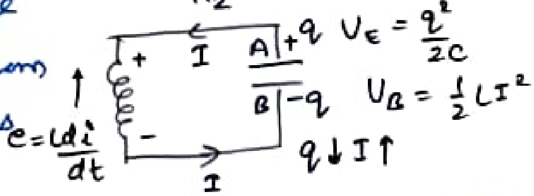


Same energy from the cell is stored in the dielectric md. b/w the plates of capacitor in the form of electrostatic energy. ($U_E = \frac{q^2}{2C}$). On removing plug of K_1 & putting in plug of K_2, the charged capacitor is connected to L & starts discharging thro' L. An induced emf opposes the growth of current in L & delays it. when the capacitor is completely discharged, the energy stored in the capacitor appears in the form of magnetic energy around L. ($U_B = \frac{1}{2}LI^2$)

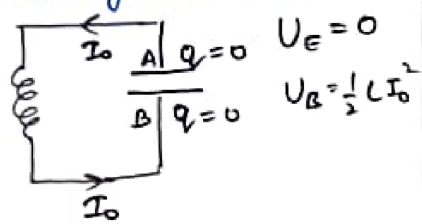
(i) Initially, the capacitor is with max^m charge, q_0 & no current in the circuit. Battery is removed & K_2 is closed.



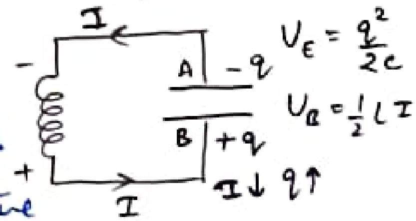
(ii) On closing the key, K_2, the plates A & B are connected with each other. The electrons from plate B move towards A, which constitutes current in opposite direction. As induced emf opposes the change in I. ∴ I will inc slowly not instantaneously & $q\downarrow$



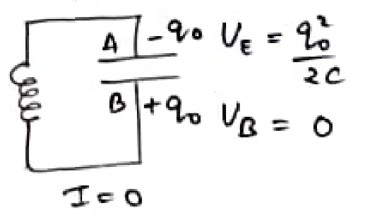
(iii) A stage will reach, which capacitor is totally discharged ($q=0$), current reaches its max^m value, I_0



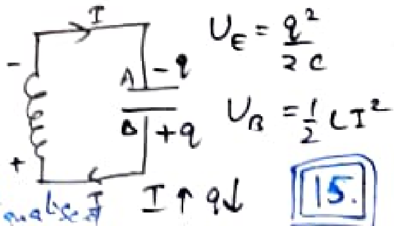
(iv) As the capacitor is totally discharged & there is no flow of current, current tends to decrease (to I, say), e.m.f induced in the coil would oppose the decrease in I



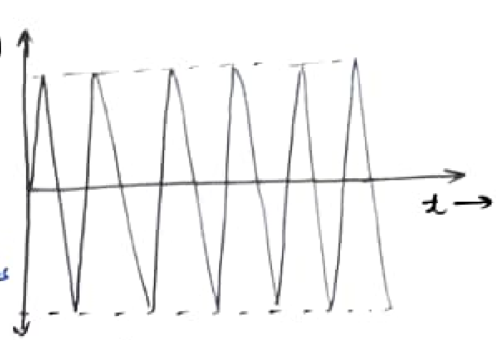
(v) Soon plate B will acquire +ve charge & plate A will attain equal −ve charge. The capacitor gets charged again but with opposite polarity.



(vi) A has excess e^-, movement of e^- again starts now from plate A to B, setting flow of current from B to A. The flow of e^- again starts till the pot are equalised

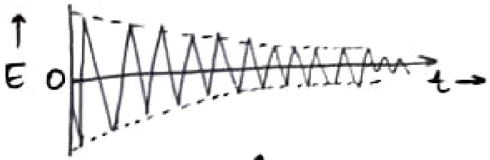


- Energy oscillates b/w Electrostatic energy of capacitor & magnetic energy around Inductor.

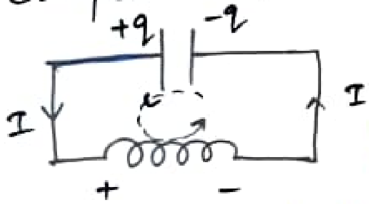


- If the circuit has no Resistance, No loss of energy would occur. The oscillations produced will be of constant amplitude. These oscillations are called Undamped Oscillations.
- If there is no loss of energy, T.E. must be constant.

$$T.E. = \underset{(I=0)}{\frac{q_0^2}{2C}} = \underset{(q=0)}{\frac{1}{2}LI_0^2} = \frac{q^2}{2C} + \frac{1}{2}LI^2$$



- In actual practice, there do occur some losses of energy. These are called
∴ Amplitude of oscillations goes on decreasing.
damped Oscillations.



$$\frac{q}{C} - \frac{LdI}{dt} = 0 \quad \left[\text{Kirchoff's loop Rule} : \Sigma\Delta V = 0\right]$$

As $q\downarrow$, $I\uparrow$ ∴ $I = -\frac{dq}{dt}$

$$\therefore \frac{q}{C} - L\left(-\frac{d^2q}{dt^2}\right) = 0 \Rightarrow \boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$

This eqn has the form $\frac{d^2x}{dt^2} + \omega^2 x = 0$, for a simple harmonic oscillator.

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi\nu \quad \therefore \boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$$

for $\frac{d^2x}{dt^2} + \omega^2 x = 0 \xrightarrow[\text{solution}]{\text{General}} x = x_0 \sin(\omega t + \phi)$
max displacement

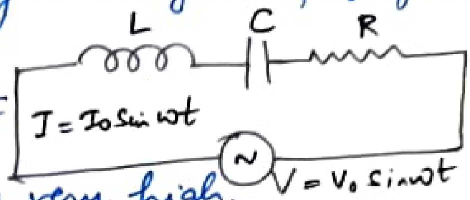
Ily, for $\frac{d^2q}{dt^2} + \frac{q}{LC} = 0 \longrightarrow q = q_0 \sin(\omega t + \phi)$ $\omega = \frac{1}{\sqrt{LC}}$
max charge

at $t = 0$, $q = q_0$, ∴ $q_0 = q_0 \sin\phi$
$\Rightarrow \phi = 90^\circ \Rightarrow \underline{q = q_0 \sin(\omega t + 90^\circ) = q_0 \cos\omega t}$

RESONANCE IN L-C-R CIRCUIT :- If a system has a natural tendency to oscillate at a particular freq. (natural freq. of oscillations), & such a system is driven by an energy source, whose frequency is equal to the natural freq of the system, the amplitude of oscillations become large & resonance is said to occur.

SERIES RESONANCE CIRCUIT:-

A circuit in which inductance L, Capacitance C & resistance, R are connected in series and the circuit admits max^{m} current corresponding to a given freq of a.c. is called Series Resonance Circuit.



As, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At low frequencies, X_L is negligible & X_C is very high.

As freq. of alternating emf applied to the circuit is increased, X_L goes on increasing & X_C goes on decreasing.

for a particular value of ω, $X_L = X_C$

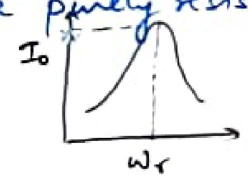
$$\Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{\nu_r = \frac{1}{2\pi\sqrt{LC}}} \quad (\because \omega = 2\pi\nu)$$

ν_r : Resonance frequency

for $X_L = X_C$, $Z = \sqrt{R^2} = R$ = Minimum

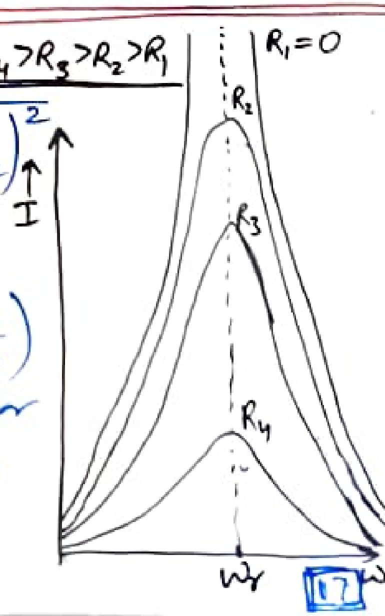
As Z is minimum, hence current is max^{m}, $I_0 = \frac{V_0}{Z} = \frac{V_0}{R}$



* At Resonance, Series LCR circuit is equivalent to a purely resistive circuit. Hence, Current & Voltage are in phase.
* The freq. (ν_r) of the applied voltage for which the current in RLC circuit b'comes max^{m} is called the resonant frequency.
* for a freq, ν greater or less than ν_r, I_0 has values less than $(I_0)_{max}$.
* As, $\tan\phi = \frac{(X_L - X_C)}{R} = 0 \Rightarrow \phi = 0°$

{SRC admits I_0 at resonance thro' it ∴ called Acceptor circuit. used in radio and T.V. receiver sets.}

SHARPNESS OF RESONANCE OF A SERIES RLC CIRCUIT: Q-FACTOR:



As, $I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

At $\omega = \omega_r$, $I_0 = \frac{V_0}{R}$

& $P_{av} = \frac{V_0 I_0}{2}\cos\phi = \frac{V_0^2}{R}$ $\left(\because \phi = 0°, I_0 = \frac{V_0}{R}\right)$

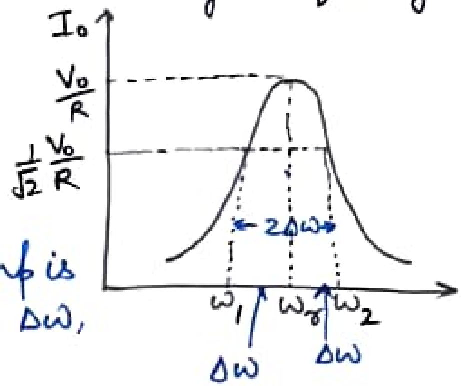
The more quickly the current amplitude (I_0) falls for changes of freq. on both sides of resonant freq ω_r the sharper is said to be the Resonance.

The sharpness of resonance of a circuit is described by its quality factor, Q (Q - factor)

$$Q = \frac{\omega_r}{\omega_2-\omega_1} = \frac{\omega_r}{2\Delta\omega}$$

$$\Delta\omega = \omega_r - \omega_1$$
$$\Delta\omega = \omega_2 - \omega_r$$
$$\underline{2\Delta\omega = \omega_2 - \omega_1}$$

bandwidth of the circuit



ω_1 & ω_2 : Values of ω for which the current amp is $1/\sqrt{2}$ times its max^m value. Smaller the value of $\Delta\omega$, the sharper or narrower is the resonance.

As $$\frac{V_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}}\frac{V_0}{R}$$

$$\therefore \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\Rightarrow \left(\omega L - \frac{1}{\omega C}\right) = \pm R \; : \; \left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R \quad (1) \quad \& \quad \left(\omega_2 L - \frac{1}{\omega_2 C}\right) = +R \quad (2)$$

(1) + (2) gives : $$(\omega_1+\omega_2)L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \frac{\omega_1+\omega_2}{(\omega_1\omega_2)C}$$

$$\Rightarrow \omega_1\omega_2 = \frac{1}{LC} = \omega_r^2 \quad -(3)$$

$$\Rightarrow \boxed{\omega_r = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1\omega_2}}$$

The Resonant freq., ω_r is thus the geometric mean ($\sqrt{\omega_1\omega_2}$) of the upper half & lower half power frequencies.

(2) - (1) gives : $$(\omega_2-\omega_1)L + \frac{(\omega_2-\omega_1)}{(\omega_1\omega_2)C} = 2R \quad -(4)$$

combining (3) & (4) : $$(\omega_2-\omega_1) = \frac{R}{L}$$

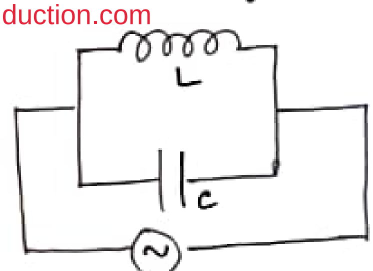
As, $$Q = \frac{\omega_r}{\omega_2-\omega_1} = \frac{\omega_r}{R/L} = \frac{\omega_r L}{R} = \frac{X_L}{R}$$

Q factor thus, also be defined as the ratio of reactance (inductive or capacitive) at resonance to the resistance of the circuit

Since, at Resonance, $X_L = X_C$: $$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{1}{\omega_r C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

* Q is only a number, having no dimensions.
* At the receiving station, many radio signals are often present over a range of frequencies. In order to eliminate unwanted signals, we need to design high Q value circuit.

PARALLEL RESONANCE CIRCUIT :-



$V = V_0 \sin\omega t$

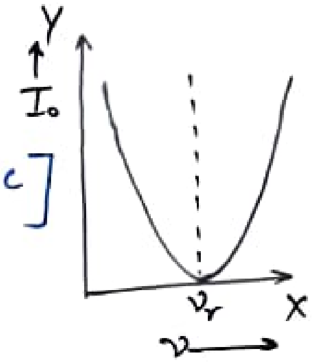
$$I_L = \frac{V_0}{X_L}\sin(\omega t - \pi/2), \quad I_C = \frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

Total Current, $I = I_L + I_C$

$$= \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow I = \frac{V_0}{X_L}(-\cos\omega t) + \frac{V_0}{X_C}\cos\omega t$$

$$= V_0\cos\omega t\left[-\frac{1}{X_L} + \frac{1}{X_C}\right] = V_0\cos\omega t\left[-\frac{1}{\omega L} + \omega C\right]$$



$\underline{I = 0}$, when $\underline{\omega C - \frac{1}{\omega L} = 0}$

$$\Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}} \Rightarrow \nu = \frac{1}{2\pi\sqrt{LC}}$$

* In parallel L-C circuit, At Resonance, Circuit doesn't allow any current to flow thro' it. ∴ Impedance must be max^m.
* The parallel resonance circuits are used in transmitting circuits. They reject the currents corresponding to parallel resonance frequencies, & allow other freq. to pass thro'. Such circuits are therefore called filter circuits or rejector circuits or even anti-resonance circuits in a communication system.

TRANSFORMER :-

Voltage Regulators, Induction furnaces, welding purposes - step down, In transmission of ac over long distances

Uses of Transformer

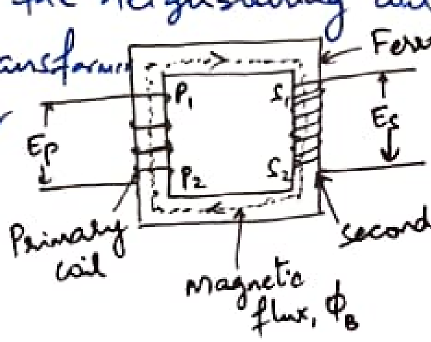
A Transformer is an electrical device, which is used for changing the a.c. voltages.

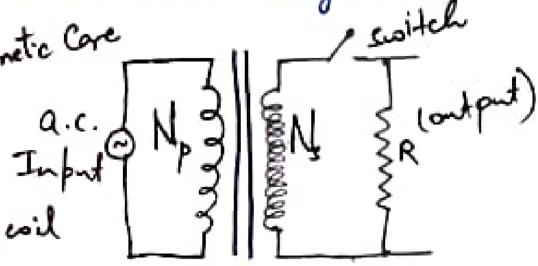
Step-up transformer: A transformer which increases the a.c. voltages is called step-up transformer.

Step-down transformer: A transformer which decreases the a.c. voltages is called step-down transformer.

Principle :- A transformer is based on the principle of mutual Induction i.e., whenever the amt of mag. flux linked with a coil changes, an emf is induced in the neighbouring coil.

Construction :- A transformer consists of rectangular soft iron core made of laminated sheets well insulated from each other.





$P_1P_2 \rightarrow$ Primary coil, $S_1S_2 \rightarrow$ Secondary coil. A source of alternating e.m.f is connected to pri coil & a load resistance, R is connected to secondary coil.

* for an ideal transformer → R of pri & sec. windings are negligible
→ Energy losses (hysteresis) are negligible

Theory & Working:- When an alternating e.m.f. $\mathcal{E}_p$ is applied to the pri. coil, the strength of current in the pri coil changes. Therefore, the mag. flux linked with the core of the transformer undergoes a change. This results in an induced emf in the primary as well as in the sec. coils. If we assume that there is no loss of mag. flux, the induced emf in the primary must be equal to the inducing alternating emf applied to it.

$$\mathcal{E}_p = -N_p \frac{d\phi_B}{dt}$$

$\mathcal{E}_p \to$ Induced e.m.f in the pri coil

$N_p \to$ no. of turns in the primary.

$$\& \quad \mathcal{E}_s = -N_s \frac{d\phi_B}{dt}$$

$\mathcal{E}_s \to$ emf induced in the secondary at the same instant

$N_s \to$ no. of turns in the secondary.

$$\therefore \boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p}}$$

For a given transformer, $\frac{N_s}{N_p} = \text{constant} = K$

$K \to$ Transformation Ratio

Step-up Transformer $\to$ Votage is raised, $\mathcal{E}_s > \mathcal{E}_p \therefore N_s > N_p \Rightarrow K > 1$

Step-down Transformer $\to$ Voltage is lowered, $\mathcal{E}_s < \mathcal{E}_p \therefore N_s < N_p \Rightarrow K < 1$

For an ideal transformer: Input power = Output Electrical Power.

$$\mathcal{E}_p I_p = \mathcal{E}_s I_s \Rightarrow \boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{I_p}{I_s}}$$

Clearly, If $\mathcal{E}_s \uparrow \Rightarrow I_s \downarrow$. When output voltage is large, the current in the output is small. The Reverse is true for step-down transformer.

$$\text{As, } I_p = I_s\left(\frac{N_s}{N_p}\right) = \frac{\mathcal{E}_s}{R}\left(\frac{N_s}{N_p}\right) = \frac{1}{R}\mathcal{E}_p\left(\frac{N_s}{N_p}\right)\left(\frac{N_s}{N_p}\right) = \underline{\frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 \mathcal{E}_p}$$

$$\therefore \frac{\mathcal{E}_p}{R_{eq}} = \frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 \mathcal{E}_p \Rightarrow \boxed{R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R}$$

load R as seen by the source/generator

Efficiency of a transformer:- $\eta = \frac{P_o}{P_i} = \frac{\mathcal{E}_s I_s}{\mathcal{E}_p I_p}$

For an ideal transformer, ∄ no power loss & $\eta = 100\%$ (Practically impossible)

Energy losses in Transformer:- (i) Cu loss (form of heat-minimised → thick wires) (ii) Fe loss (Heat → form of eddy currents → minimised → laminated cores) (iii) leakage of ϕ_B → reduced by winding the pri & sec. coils one over the other. (iv) Hysteresis loss (v) Magnetostriction (humming noise of a transformer)