PHYSICS INDUCTION
An institute of Science & Mathematics

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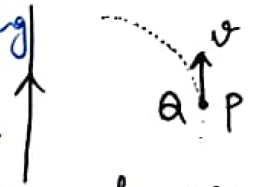
CLASS XII : NOTES : CHAPTER-4 : MOVING CHARGES AND MAGNETISM : PHY

NATURE OF MAGNETISM :-

Charges at rest : $\vec{F}_E$ (Electric Force)

Charges in motion → $\vec{F}_E$ (Electric force) → depends on Q
→ $\vec{F}_B$ (Magnetic Force) → depends on both Q and v(vel)

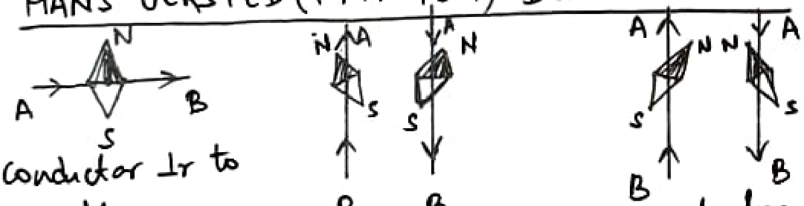
For a charge, Q placed at rest near a current carrying wire, there is no appreciable electric field at pt., P. ∵ in any volume of wire, there are equal amt of +ve & -ve charges. The wire is electrically neutral ⇒ No $\vec{E}$



Current carrying wire

If the charge, Q is projected from the pt, P in the directⁿ of current, it is deflected towards the wire (Q - +ve) - assumption. There must be a field ⇒ MAGNETIC FIELD.

HANS OERSTED (1777-1851) - DEMONSTRATION : He noticed that current in a straight wire caused a noticeable deflection in a nearby magnetic compass needle. Reversing the directⁿ of current, reverses the orientation of the needle. The deflection increases on increasing the current or bringing the needle closer to the wire.



Conductor ⊥r to needle

Conductor above needle

conductor below needle

* Iron filings sprinkled around the wire arrange themselves in concentric circles, with the wire as the centre.

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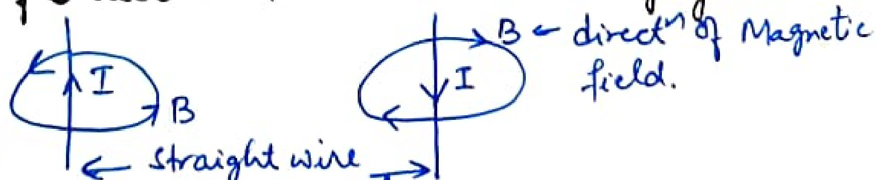
* Convention
- ⊙ : I or field emerging out of the plane
- ⊗ : I or field going into the plane.

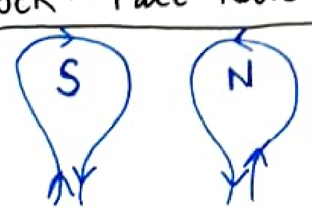
RULES : To find the directⁿ of B associated with a current carrying conductor.

1. Right Hand Thumb Rule :-

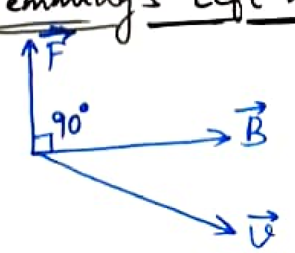
Thumb - directⁿ of current & fingers will wrap around the conductor in the direction of magnetic field.



2. Clock - Face Rule :-



3. Flemming's Left hand Rule :-

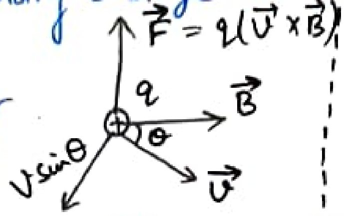


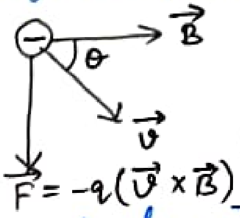
(To find the directⁿ of F experienced by a moving q, placed in a mag. field)

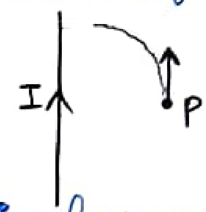
MAGNETIC LORENTZ FORCE :- $\vec{F}_g = m\vec{g}$, $\vec{F}_E = q\vec{E}$

If we place a stationary charge in a magnetic field, the charge experiences no force.

If $\vec{v}$ & $\vec{B}$ are parallel or anti parallel, No force is experienced due $\vec{B}$.







Experimentally, it has been found that mag. of force, $\vec{F}$ experienced by the moving charge, : (a) $\vec{F} \propto q$

(b) $\vec{F} \propto v\sin\theta$ (component of v acting ⊥r to $\vec{B}$)

& (c) $\vec{F} \propto B$

$F \propto qvB\sin\theta$

$\Rightarrow F = KqvB\sin\theta$ (But $K=1$)

$\Rightarrow |\vec{F}| = q|\vec{v}\times\vec{B}|$ or $\vec{F} = q(\vec{v}\times\vec{B})$

If $v=1, q=1$ & $\sin\theta = 1$ or $\theta = 90°$ then $F = B$

magnetic field induction or Magnetic flux density at a point in the mag. field is equal to the force experienced by a unit charge moving with a unit velocity ⊥r to the direct^n of B at that pt.

*** Special cases :-**

Case 1. $\theta = 0°$ or $180°$, $F = qvB = 0$

Case 2. $\theta = 90°$ ∴ $F = qvB$ (Maximum)

Case 3. $\theta = \frac{3\pi}{2} = 270°$, $F = -qvB$

Case 4. If $v = 0$, $F = 0$

Unit of $\vec{B}$:- S.I. Unit of B is tesla (T) // weber/(meter)^2 $[Wb/m^2]$ // $N s C^{-1} m^{-1}$

1 Tesla :- $B = \frac{F}{qv\sin\theta}$ ⇒ $F = 1N$, $v = 1m/s$, $\theta = 90°$, $q = 1C$ ⇒ $B = 1T$

∴ B at a pt is said to be 1T, if a charge of 1C while moving at a st. angle to a magnetic field, with a velocity of 1m/s, experiences a force of 1N, at that point.

{* Earth's Mag. field → 3.6×10^{-5} T }

C.g.s unit of B :- Guass, G , $1G = 10^{-4}T$

Dimensions of $\vec{B}$: $B = \frac{F}{qv\sin\theta}$ ∴ $[B] = \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [MA^{-1}T^{-2}]$

LORENTZ FORCE :-

The force experienced by a charged particle moving in space is due to both the electric & magnetic field. $\vec{F}_E$ is ind. of motion of q.

∴ Total Force, $\vec{F} = \vec{F}_E + \vec{F}_m$

$$\Rightarrow \underset{\text{Lorentz force.}}{\vec{F}} = q\vec{E} + q(\vec{v}\times\vec{B}) = q[\vec{E} + \vec{v}\times\vec{B}]$$

FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD :-

Consider a rod of uniform cross-sectional area, A and length, l.

let n be the number density of mobile charge carriers (e^-)

∴ Total no. of e^- = $(Al).n$

For steady current, I, we may assume that each e^- has an average drift velocity, $\vec{v}_d$

$\therefore$ Force acting on each e^- due to $\vec{B}$, $\vec{f} = -e(\vec{v}_d \times \vec{B})$

Total Force acting on all the electrons, $\vec{F} = (Aln)\vec{f}$

$\Rightarrow \vec{F} = (Aln)[-e(\vec{v}_d \times \vec{B})]$

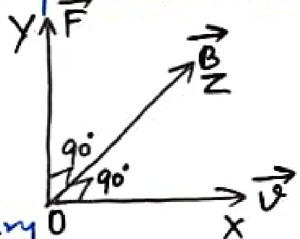
$= (-neAl\,\vec{v}_d) \times \vec{B}$

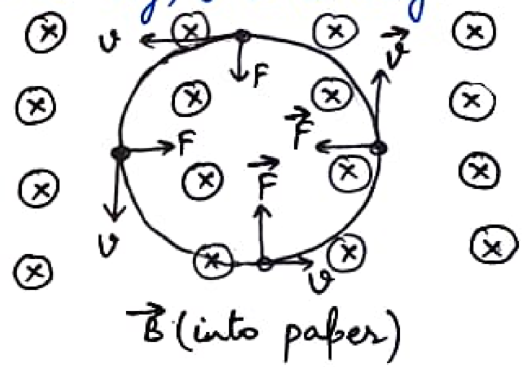
$= I\vec{l} \times \vec{B}$

$\left\{ \because I = AneV_d \Rightarrow Il = AneV_d . l \right.$
$\because \vec{l}$ & $\vec{v}_d$ are in opp. directions.
$\left. \therefore I\vec{l} = -neAl\,\vec{v}_d \right\}$

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B}) = q\left(\frac{\vec{l}}{t} \times \vec{B}\right) = \frac{q}{t}(\vec{l} \times \vec{B}) = I(\vec{l} \times \vec{B})}$$

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD:- Consider a beam of positively charged particles, moving with velocity, $\vec{v}$ in a mag. field, $\vec{B}$. $\vec{v} \perp \vec{B}$, $\theta = 90^\circ$





$\therefore F = qvB$

$\because \vec{F} \perp \vec{v}$ (always)

$\therefore$ it will provide necessary Centripetal force for the charged particle to move in a circular path of radius, r.

$\therefore F_c = \frac{mv^2}{r}$, m: mass of the particle

$\Rightarrow qvB = \frac{mv^2}{r} \Rightarrow \boxed{r = \frac{mv}{qB}}$ r: gyroradius or cyclotron radius. $\underline{r \propto v}$

$\therefore \frac{v}{r} = \frac{qB}{m}$

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$\Rightarrow \omega = \frac{qB}{m} \Rightarrow 2\pi\nu = \frac{qB}{m} \Rightarrow \boxed{\nu = \frac{qB}{2\pi m}}$ ν: gyrofrequency or cyclotron frequency $\underline{\nu \propto B}$

* As $F = q(\vec{v} \times \vec{B})$ & $F \perp v \Rightarrow F \perp \frac{ds}{dt} \Rightarrow F \perp ds$

or $dW = \vec{F}.d\vec{s} = 0 \Rightarrow$ F does no work

* As, F does no Work $\Rightarrow$ No change in K.E $\Rightarrow |\vec{v}|$ remains constant.

* As direct^n of $\vec{v}$ changes $\therefore \vec{p} = m\vec{v}$ changes.

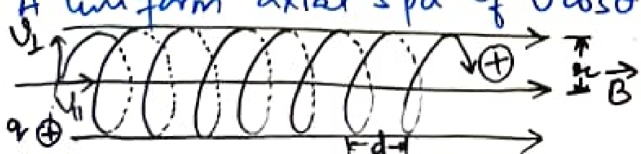
* Mag. F simply changes the direct^n of $\vec{v}$ $\therefore$ called deflecting force.

* If $\vec{v}$ makes an angle, θ (other than 90°) with $\vec{B}$, then the particle will move in a helix. The axis of helix is parallel to $\vec{B}$. The helical motion is a result of superposing:

→ A uniform circular motion in which part has a spd $v\sin\theta$.$(=v_\perp)$ $\underline{\perp B}$

→ A uniform axial spd of $v\cos\theta\,(=v_\parallel)$ along the direct^n of $\vec{B}$.

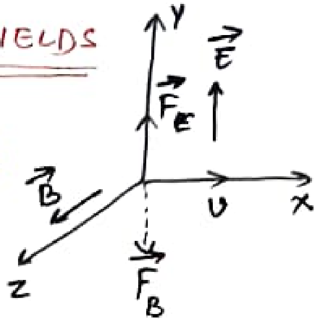
pitch: linear dist. covered in 1 rotation



* $d \rightarrow$ pitch

$d = v\cos\theta \times T = v\cos\theta \times \frac{1}{\nu} = \frac{v_\parallel}{\nu}$

MOTION IN COMBINED ELECTRIC AND MAGNETIC FIELDS



• **Velocity selector**:- As $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Consider $\vec{E} \perp \vec{B} \perp \vec{v}$, $\vec{v} = v\hat{i}$, $\vec{E} = E\hat{j}$, $\vec{B} = B\hat{k}$

$\therefore \vec{F}_E = q\vec{E} = qE\hat{j}$

& $\vec{F}_B = q(v\hat{i} \times B\hat{k}) = -qvB\hat{j}$

$\therefore \vec{F} = q(E - vB)\hat{j} \Rightarrow F_E$ & F_B are in opp. directⁿ.

let's adjust the value of $\vec{E}$ & $\vec{B}$ s.t., mag. of the two forces are equal. then, net F on a given charge = 0 & the charge will move in the field undeflected.

if $F_E = F_B$

$\Rightarrow qE = qvB$

$\Rightarrow \boxed{v = \frac{E}{B}}$

* Only particles with the speed E/B pass undeflected through the region of crossed fields.
* The crossed E & B fields ∴ serve as velocity selector.

• **Cyclotron**:- A cyclotron is a device used for accelerating positively charged particles like proton, deuteron, alpha particles and heavier positive ions to desired velocities. It was developed by Lawrence and Livingstone.

Principle:- (i) It's based on the fact that a positively charged particles can be accelerated to a sufficiently high energy with the help of smaller values of oscillating electric field by making it to cross the same electric field time and again, with the use of strong magnetic field.

* To accelerate → high p.d. (several million volts) is reqd → can't be created in labs. Apparatus is so designed that the particles are subjected to smaller p.d. several times in succession.
* When a charged particle of mass, m and charge, q moves perpendicular to a uniform $\vec{B}$, then, it has a frequency, $\nu_c = \frac{qB}{2\pi m}$

ν_c : cyclotron frequency

ν_c is independent of the speed, v and radius, r of the path of the particle.

Construction:- A cyclotron consists of the following parts :

(a) Two D-shaped hollow evacuated metal chambers D_1 & D_2 called dees. These dees are placed horizontally with their diametric edges parallel and slightly separated from each other.

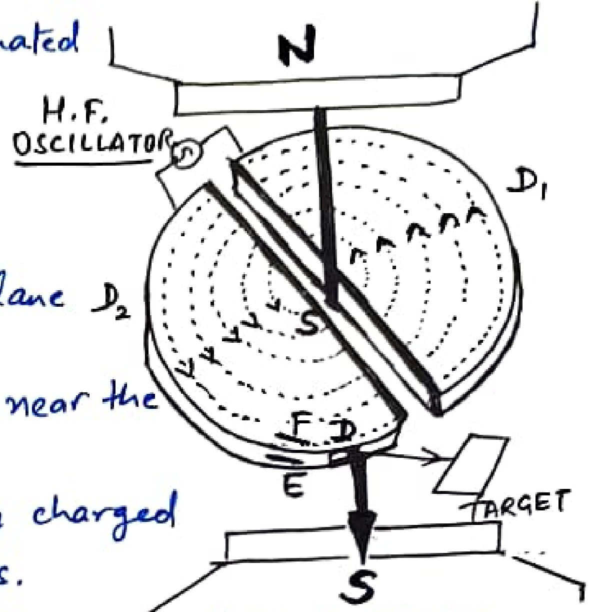
(b) The dees are connected to a high frequency oscillator which can produce a potential difference of the order of 10^4 V at frequency 10^7 Hz.

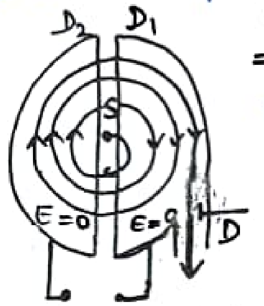
(c) The two dees are enclosed in an evacuated steel box & are well insulated. Vacuum chamber is evacuated to a pressure of 10^{-6} mm of Hg → to prevent the ions from colliding with the air molecules.

(d) NS is a strong magnet placed in a plane ⊥r to the plane of the dees.

(e) S is an ion(+ve)-source, which is located near the midpoint of the gap b/w the two dees.

(f) A deflector plate, D, which is negatively charged is used to pull the ions out of the dees.





High frequency Oscillator

Working and Theory:- When a positive charge is emitted from the source, S, At any instant, suppose D_1 is at −ve potential & D_2 is at +ve potential. Therefore, the positively charged ion produced at S, will be accelerated towards D_1. On reaching inside D_1, the ion will be in a field free space. Hence, it moves with a const velocity in D_1 (say v). But due to mag. field (⊥r), the ion will describe a circular path of radius, r in D_1.

$$\therefore \quad qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{Bq}$$

time taken by the ion to describe a semicircular path is given by,

$$t = \frac{\pi r}{v} = \frac{\pi m}{Bq} = \frac{\pi}{B(q/m)} = \text{a Constant}$$

* The frequency of the oscillator is so adjusted that the time taken by the particle to describe a semicircular path is equal to the time taken by the oscillator to complete half cycle.

* As, time, t is independent of v and r of the circle in which it moves, time taken by a particle to travel in a semicircle has been made equal to the time taken by the dees to change their polarities, the particle will be continuously accelerated & will move along semicircular paths of increasing radii. When it comes out, its velocity will be very high & as such it will possess a large K.E.

T is the time period of the high frequency oscillator, $t = T/2$

$$T = 2t = \frac{2\pi m}{qB}$$

The frequency of the oscillator (ν_{osc}) is given by

$$\nu_{osc} = \frac{1}{T} = \frac{qB}{2\pi m}$$ ← Magnetic Resonance frequency

Maximum Energy of positive ion:- If v_0 : Max Vel , r_0 : max radius

Then, $\frac{mv_0^2}{r_0} = Bqv_0 \Rightarrow v_0 = \frac{Bqr_0}{m}$

$\Rightarrow$ Max K.E. = $\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{Bqr_0}{m}\right)^2 = \frac{B^2q^2r_0^2}{2m}$

Functions of Electric and Magnetic fields in a Cyclotron:- Electric field is used to accelerate heavy charged particles in a region b/w the two dees. The magnetic field makes the heavy charged particle to describe the circular path inside the dees.

Limitations of Cyclotron:-

- When the ion is accelerated, it moves with greater & greater speed. As the speed of the ion b'comes comparable with that of light, the mass of the ion increases. $\left(m = m_0/\sqrt{1-v^2/c^2}\right)$

As, $t = \frac{\pi m}{Bq} = \frac{\pi}{Bq} \cdot \frac{m_0}{\sqrt{1-v^2/c^2}}$

As $v\uparrow$, $t\uparrow$ $\Rightarrow$ +ve ion will take longer time to describe semicircular path. As a result of it, the ion will not arrive in the gap b/w the 2 dees exactly at the instant, the polarity of the 2 dees is reversed & hence, will not be accelerated further.

- It's suitable for accelerating heavy particles like proton, deutron α-part. etc.
- The Uncharged part (neutrons) & lighter charged part (electrons) can't be accelerated by cyclotron.

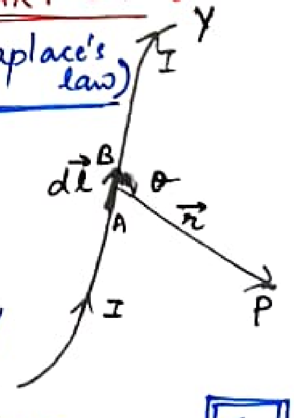
Uses of Cyclotron:- The high energy charged particles are —

- used to bombard the atomic nuclei.
- used to produce other high energy particles (like neutrons) thro' bombardment process, which are used in atomic reactors.
- used for artificial transmutation in order to produce radio isotopes which are used for medical purposes.

MAGNETIC FIELD DUE TO A CURRENT ELEMENT — BIOT-SAVART LAW:-

(Also called as Laplace's law)

It's an experimental Law, which deals with the magnetic field at a point due to a small current element (i.e., small part of any conductor carrying current.



Consider a small element $AB(=dl)$ of cond, XY carrying current, I. Let $\vec{r}$ be the posn vector of the pt., P from the current element, $I d\vec{l}$.

θ : $\angle$ b/w $d\vec{l}$ & $\vec{r}$

∴ Mag. field Induction (dB) at a pt., P due to current element depends upon the following factors: $dB \propto I$, $dB \propto dl$, $dB \propto \sin\theta$, $dB \propto 1/r^2$

$$\therefore dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow dB = K\frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

where, $K = \frac{\mu_0}{4\pi}$ (In SI Units) & $K = 1$ (In cgs system)

μ_0: Absolute Mag. permeability of free space $= 4\pi\times10^{-7}\,WbA^{-1}m^{-1} = 4\pi\times10^{-7}\,TA^{-1}m$

In Vector form: $|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I|d\vec{l}\times\vec{r}|}{r^3}$ or $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(d\vec{l}\times\vec{r})}{r^3}$

Direction of $d\vec{B}$:

Magnetic field induction at pt., P due to current thro' entire wire :-

$$\vec{B} = \int\frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^3} = \int\frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

In terms of j: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{J}\times\vec{r}}{r^3}dV$ $\quad\left(\because j = \frac{I}{A} = \frac{I\,dl}{A\,dl} = \frac{I\,dl}{V}\right)$

In terms of q & v:- $d\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ $\quad\left(\because I\,d\vec{l} = \frac{q}{dt}\cdot d\vec{l} = q\vec{v}\right)$

* Valid for symm. current distⁿ, applicable only to very small length cond.
* analogous to Coulomb's law in Electrostats.
* $\theta = 0°$ or $180°$, P → on the axis of conductor, $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 0°}{r^2} = 0$
  $dB \to$ Minimum.
* $\theta = 90°$, P → lies at a ⊥r posⁿ, $dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$ ∴ $dB \to$ Maximum.

BIO-SAVART LAW Vs COULOMB'S LAW:

* long range, depend on $1/r^2$, pri of superposⁿ applies to both.
* $\vec{E} \to$ Scalar source → charge, $\vec{B} \to$ Vector source → $I\,d\vec{l}$
* $\vec{E}$ – along $\vec{r}$, $\vec{B}$ ⊥r to $\vec{r}$ & $I\,d\vec{l}$
* No angle dependence in $\vec{E}$, $d\vec{B} = 0$ at $\theta = 0°, 180°$

Relationship b/w μ_0 & ε_0:-

$$\varepsilon_0\mu_0 = (4\pi\varepsilon_0)\left(\frac{\mu_0}{4\pi}\right) = \frac{1}{(9\times10^9)}(10^{-7}) = \frac{1}{(3\times10^8)^2} = \frac{1}{c^2}$$

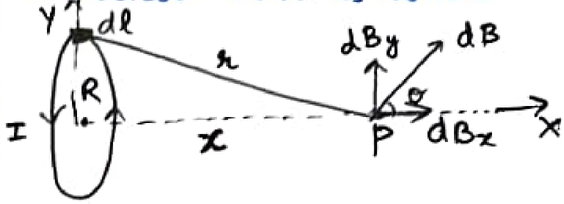
$$\Rightarrow \boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}}$$

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP :-

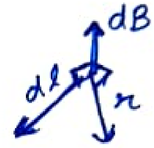
Consider a circular loop of radius, R, carrying a steady current, I, placed in the y-z plane, with its centre at origin.

Let P be the point, at a distance, x from centre of the loop, where magnetic field due to current carrying circular coil is to be measured.

Consider a conducting element, dl of the loop. The magnitude dB of the magnetic field due to dl :



$$dB = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3} \quad \{|d\vec{l} \times \vec{r}| = r\,dl\}$$

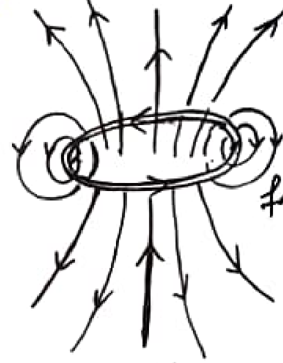


$$= \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \quad (\text{But, } r^2 = x^2 + R^2)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{(x^2+R^2)}$$

y-component of dB are equal and opposite, they cancel out each other & the x-components, being in the same direct^n are added up.

$$\therefore \; dB_x = dB\cos\theta$$



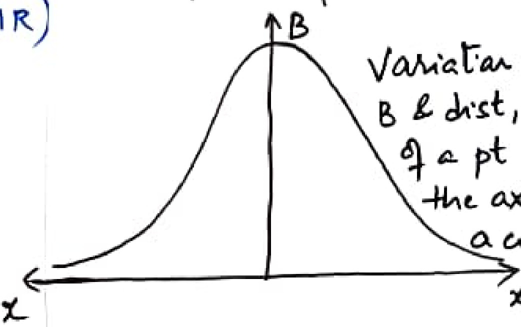
magnetic field lines for a current loop.

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{x^2+R^2} \times \frac{R}{(x^2+R^2)^{1/2}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \cdot R}{(x^2+R^2)^{3/2}}$$

$$B = \int dB\cos\theta = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot R}{(x^2+R^2)^{3/2}} (2\pi R)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}}$$



Variation of B & dist, x of a pt on the axis of a circular coil (I)

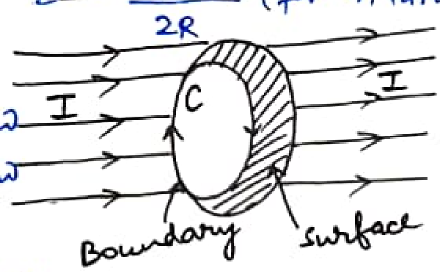
Vector Not

$$\vec{B} = B\hat{i} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}\hat{i}$$

At the centre of the loop: $x = 0$

$$\therefore B_0 = \frac{\mu_0 I}{2R}\hat{i} \qquad B = \frac{\mu_0 n I}{2R} \text{ (for n turns)}$$

AMPERE'S CIRCUITAL LAW :-



As, Guass's Law → Alternative form of Coulomb's law

Illy, Ampere's Circuital Law → " " Bio-Savart law

Ampere's Circuital law considers an open surface with a boundary. The surface has current, I passing through it. Let the boundary, C be made up of large no. of small line elements, dl. direct^n of $d\vec{l}$ is tangential to dl.

$\vec{B}_t$: tangential comp. of mag field. Then, $\vec{B}_t$ & $d\vec{l}$ are acting in the same direct^n.

$\vec{B}.\vec{dl} = B_t\, dl \cos 0^\circ = B_t\, dl$

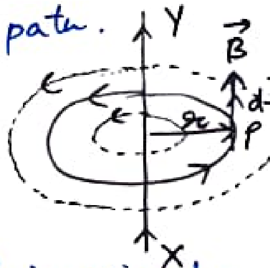
Sum of $\vec{B}.\vec{dl}$ over all elements on a closed path $= \oint \vec{B}.\vec{dl}$

Ampere's Circuital law states that the line-integral of $\vec{B}$ around the closed path is equal to μ_0 times the total current passing thro' the surface. $\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I}$

I_1 : +ve, I_2 : -ve,

$\oint \vec{B}.\vec{dl} = \mu_0 (I_1 - I_2) = \mu_0 I_e$

where, I_e : Total current enclosed by the loop or closed path.

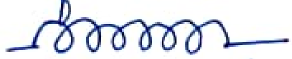


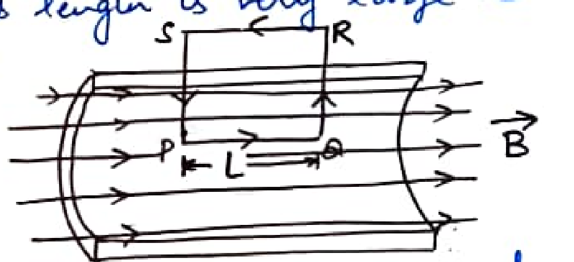
Proof of Ampere's Circuital Law :- $\vec{B}$ around a conductor → concentric circles. $\vec{B}$ is tangential to the circumference of the circle.

$\vec{B}$ at a dist., r outside the wire is tangential & is given by:

$B \times 2\pi r = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$

THE SOLENOID :- A solenoid consists of an insulating long wire closely wound in the form of helix. Its length is very large as compared to its radius.





The field produced by a solenoid is very much similar to bar magnet.

Inside the solenoid, Magnetic field is almost uniform & parallel to the axis of solenoid. B is same at all points inside. Outside the solenoid, field lines emerge from one end of the solenoid & merge at the other end.

Consider a rectangular amperian loop PQRS near the middle of solenoid, where PQ = L. Let the mag. field, B along the path PQ be B, the field along RS = 0. (QR & SP) ⊥r to the axis of solenoid, B along these paths is zero.

$$\therefore \oint \vec{B}.\vec{dl} = \int_P^Q \vec{B}.\vec{dl} + \int_Q^R \vec{B}.\vec{dl} + \int_R^S \vec{B}.\vec{dl} + \int_S^P \vec{B}.\vec{dl}$$

$$= \int_P^Q B\,dl \cos 0^\circ + \int_Q^R B\,dl \cos 90^\circ + 0 + \int_S^P B\,dl \cos 90^\circ$$

$$= BL + 0 + 0 + 0 = BL \qquad \text{— (1)}$$

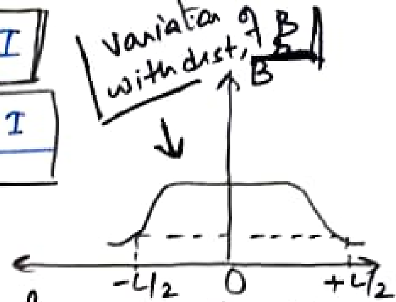
As $\oint \vec{B}.\vec{dl} = \mu_0 \times$ Total Current thro' the rectangle PQRS

$= \mu_0 \times$ no. of turns in rectangle $\times$ current

$= \mu_0 \times nL \times I$ — (2) (Total no. of turns in $L = nL$)

From ① & ②, $BL = \mu_0 n L I \Rightarrow \boxed{B = \mu_0 n I}$

N – Total no. of turns $\therefore n = N/L \Rightarrow \boxed{B = \frac{\mu_0 N I}{L}}$

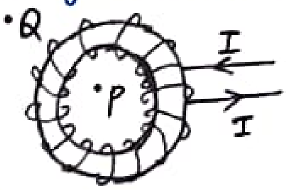


near the end of a solenoid:

$$B = \frac{\mu_0 n I}{2} = \frac{\mu_0 N I}{2L}$$

If a magnetic material of permeability, μ is filled inside the solenoid: $B = \mu n I = \frac{\mu N I}{L}$

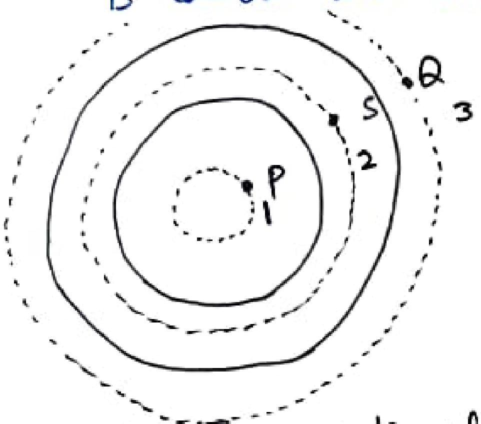
THE TOROID:- The toroid is a hollow circular ring on which a large no. of turns of a wire are closely wound.



B in the open space inside (pt. P) = 0

B exterior to the toroid (pt. Q) = 0

B inside the toroid – const. in magnitude for the ideal toroid.



Three circular Amperian loops (1,2,3) are shown by dashed lines.

B → tangential, const. in magnitude

The circular areas bounded by loops 2 & 3 both cut the toroid so that each turn of current carrying wire is cut once by the loop 2 & twice by the loop 3.

Sectional view of toroid

Loop 1 encloses no I.

Loop-1: $B_1(2\pi r_1) = \mu_0(0) \Rightarrow \underline{B_1 = 0}$

[Mag. field is zero at any pt, P inside the toroid. B at Q is likewise zero.]

Loop-3: Current coming out of the plane of the paper is cancelled exactly by the current going into it. $B_3(2\pi r_3) = \mu_0(0) \Rightarrow \underline{B_3 = 0}$

Loop-2: At S: $L = 2\pi r$

Current enclosed, $I_e = NI$ (for N turns of toroidal coil)

$$\therefore B(2\pi r) = \mu_0 N I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

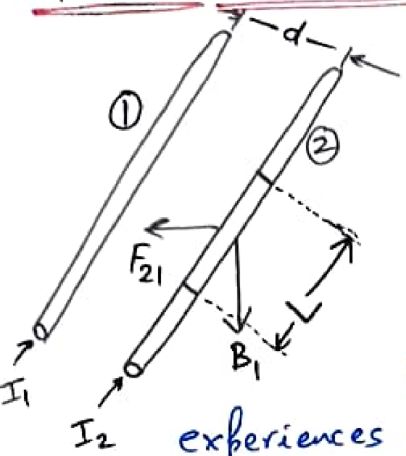
Let $r \rightarrow$ Avg. radius of the toroid, $n \rightarrow$ No. of turns per unit length

$N = 2\pi r n$

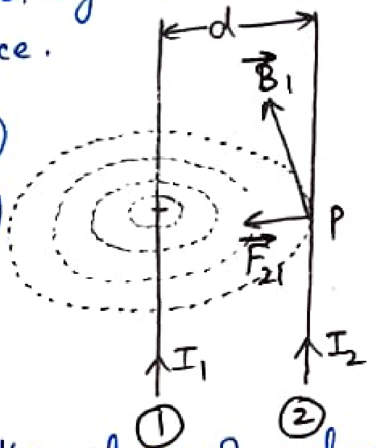
$$\therefore \boxed{B = \mu_0 n I}$$

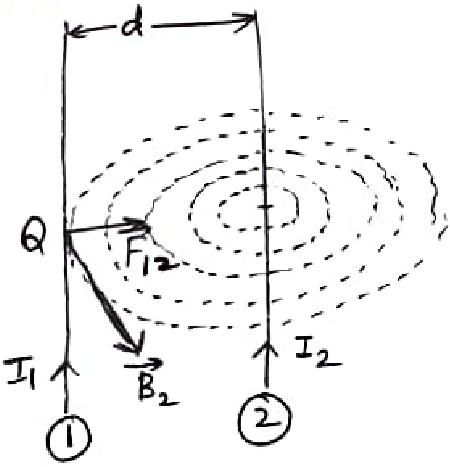
* In an ideal toroid the coils are circular. In reality, turns of the toroidal coil form a helix & there is always a small B external to the toroid.

FORCE BETWEEN TWO PARALLEL CURRENTS, THE AMPERE:



Two current carrying conductors placed near each other, exert magnetic forces on each other. Consider two infinite long straight conductors carrying currents I_1 & I_2 in the same direction, held parallel to each other at a distance, d apart. Since, each conductor is in the mag. field produced by the other, therefore, each conductor experiences a force.





B_1 at P on conductor (2) due to current, I_1 passing thro' conductor (1)

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

direct^n of $\vec{B}_1$ is ⊥r to the plane of paper, directed inwards. (right hand Rule)

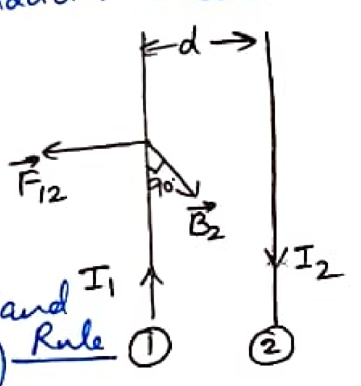
F_{21} : force on segment L of conductor 2 due to conductor 1.

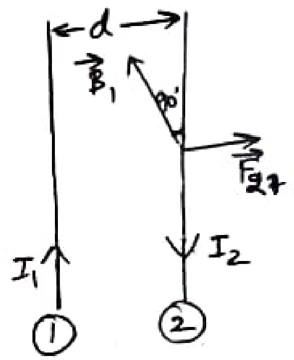
$$F_{21} = B_1 I_2 L$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \cdot L$$

Direct^n of Force - determined by Flemming's left Hand Rule

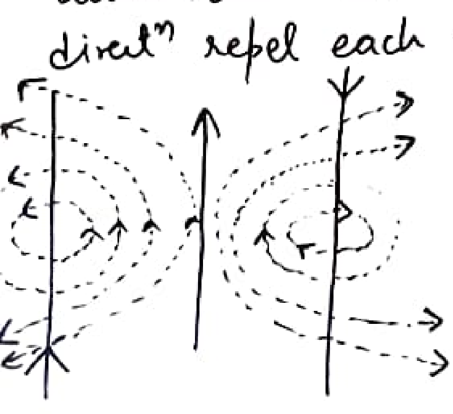
$$\vec{F}_{12} = -\vec{F}_{21} \text{ (Newton's Third Law)}$$

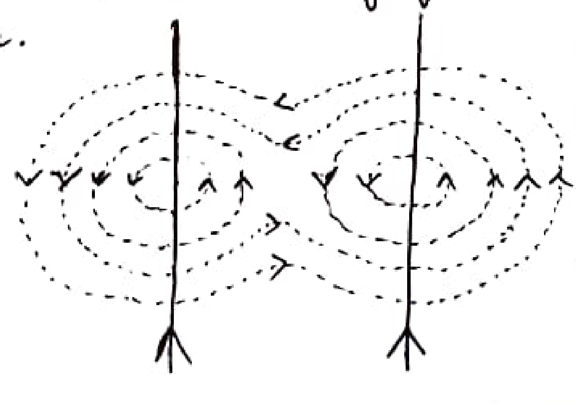




Force per unit length, $\vec{F}_2 = \frac{\vec{F}_{21}}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

* Two linear parallel conductors carrying currents in the same direct^n attract each other while conductors carrying currents in the opposite direct^n repel each other.





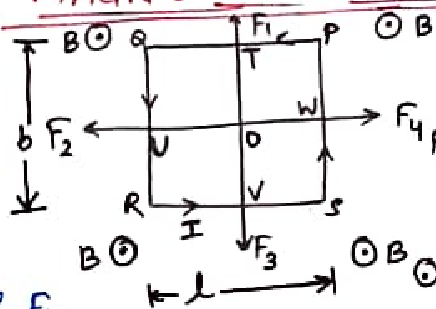
* An Instrument called the current balance is used to measure this Mechanical force.

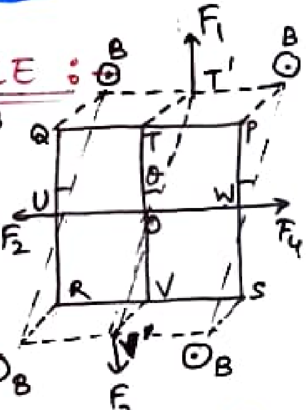
* **1 AMPERE (defⁿ - 1946):** 1A is that much current which when flowing thro' each of the two parallel linear conductors placed in free space at a dist. of 1m from each other will attract or repel each other with a force of 2×10^{-7} N/m of their length.

If, $I_1 = I_2 = 1A$, $d = 1m$

then, $F = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times1\times1}{2\pi\times1} = \underline{2\times10^{-7}\,N/m}$

TORQUE ON CURRENT LOOP, MAGNETIC DIPOLE:





Consider a rectangular loop PQRS of length, l & breadth, b carrying current, I; placed in a magnetic field, B.

The mag. forces, F_1, F_2, F_3 & F_4 on the wires PQ, QR, RS & SP are obtained by using the eqⁿ $\vec{F} = I\vec{l}\times\vec{B}$

These forces act from the middle pts T, U, V & W.

$F_1 = F_3 = IlB$ & $F_2 = F_4 = IbB$

∴ Net F = 0 (F_1 & F_3 have the same line of action, so they together produce no τ, ||ly, F_2 & F_4 together produce no τ)

Also net Torque = 0

Suppose, the loop is rotated thro' an angle, θ about the line WU

The wire PQ shifts parallel to itself with the force, $\vec{F_1} = I\vec{l}\times\vec{B}$

||ly $\vec{F_3} = I\vec{l}\times\vec{B}$

The line TV gets rotated by an angle, θ to take the posⁿ T'V'

∴ The torque of F_1 about O $= |\vec{OT'}\times\vec{F_1}| = \left(\frac{b}{2}\right)\times F_1\times\sin\theta$

$\tau_1 = \frac{b}{2}(IlB)\sin\theta$, along UW

||ly $\tau_3 = \frac{b}{2}(IlB)\sin\theta$, along UW

As, the wire QR rotates about WU, the plane containing the wire & B doesn't change. The force on the wire is ⊥r to this plane & hence, its directⁿ remains unchanged.

∴ Net Torque, $\tau = \frac{b}{2}(IlB\sin\theta) + \frac{b}{2}(IlB\sin\theta) = IlbB\sin\theta = \underline{IAB\sin\theta}$

As the loop rotates, the Area Vector, $\vec{A}$ also rotates by an angle, θ.

$$\boxed{\vec{\tau} = I\vec{A}\times\vec{B} = \vec{m}\times\vec{B}}$$

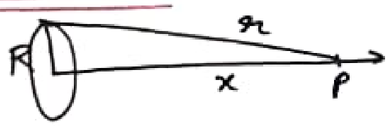
$\vec{m} = I\vec{A}$: Magnetic Dipole Moment

In $\vec{E}$: $\vec{\tau} = \vec{P}\times\vec{E}$

In $\vec{B}$: $\vec{\tau} = \underbrace{\vec{M}}_{\text{Dipole Moment}}\!\!\!= I\vec{A}\times\vec{B}$

CIRCULAR CURRENT LOOP AS A MAGNETIC DIPOLE:–

As, B due to current in a circular loop at P,



$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$ B → along the axis.

for $x >> R$, $B = \frac{\mu_0 I R^2}{2x^3}$

$$B = \frac{\mu_0 I A}{2\pi x^3} \quad (\because A = \pi R^2)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3} \quad (\because \vec{m} = I\vec{A})$$

(* Similar to $\vec{E}$ of a dipole on its axis: $\mu_0 \to 1/\varepsilon_0$, $\vec{m} \to \vec{p}$, $\vec{B} \to \vec{E}$)

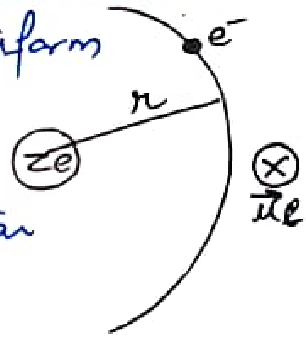
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$$

Basic difference: An electric dipole is built up of 2 elementary units - the charges (+,–), In magnetism, a mag. dipole (or current element) is the most elementary element. Magnetic monopoles don't exist.

* Current loop –
  - behaves like a magnetic dipole at large distances.
  - produces a magnetic field.
  - is subject to torque like a magnetic needle.

THE MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON:–

The electron of charge $(-e)$ $(e = 1.6\times10^{-19}C)$ performs uniform circular motion around a stationary heavy nucleus of charge, $+Ze$. This constitutes a current, I.



$I = \frac{e}{T}$, T: Time period of Revolution

$T = \frac{2\pi r}{v}$, r: Orbital radius of the e⁻, v: orbital speed.

$$\Rightarrow \quad I = \frac{ev}{2\pi r}$$

Magnetic moment (μ_e) associated with the circulating current:

$$\mu_e = IA = I(\pi r^2) = \frac{evr}{2}$$

direct^n → into the plane of paper

$$\mu_e = \frac{e}{2m_e}(m_e v r) \quad (\times \,\&\, \div \text{ by } m_e)$$

$$= \frac{e}{2m_e}\, l$$ {where, l: Magnitude of angular momentum of the e⁻ about the central Nucleus} ORBITAL ANGULAR MOMENTUM

ORBITAL MAGNETIC MOMENT

$$\Rightarrow \quad \vec{\mu}_e = \frac{-e}{2m_e}\vec{l}$$ {–ve sign : l & μ_e have opp. directions; for +q : " " " same "}

$$\left[\frac{\mu_e}{l} = \frac{e}{2m_e} = 8.8\times10^{10}\ C/kg\right]$$ μ_e/l : Gyromagnetic Ratio

Also, $l = \frac{nh}{2\pi}$ $n = 1, 2, 3, ----$
$h = 6.626 \times 10^{-34}$ Js

cond^n of discreteness ⇒ Bohr Quantisation Condition.

As, $\mu_e = \frac{el}{2m_e}$

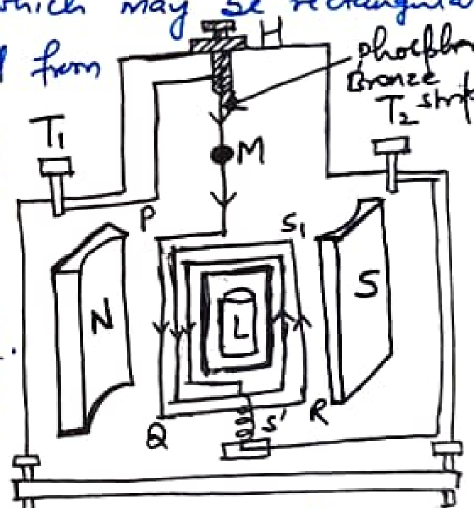
$$\Rightarrow (\mu_e)_{min}\ [\text{for } n=1] = \frac{e \cdot h}{4\pi m_e} = \frac{1.6\times10^{-19} \times 6.626\times10^{-34}}{4\times3.14\times9.1\times10^{-31}} = 9.27\times10^{-24}\ \text{Am}^2$$

This value is called Bohr Magneton. It's defined as the minimum magnetic dipole moment associated with an atom due to orbital motion of an e^- in the Ist orbit.

MOVING COIL GALVANOMETER :- It's a device used to detect feeble electric currents.

Principle :- It's based on the fact that when a current carrying coil is placed in a uniform magnetic field, it experiences a torque and it will rotate.

Construction :- It consists of two concave permanent magnet where magnetic field is directed from N to S pole. and a rectangular coil PQRS, having large no. of turns of insulated Cu wire. The coil is wound over a non-magnetic metallic frame (brass) which may be rectangular or circular in shape. The coil is suspended from a movable torsion head, H by means of a suspension fibre of phosphor bronze strip



S': Hair spring (highly elastic). The lower end of coil is connected to one end of s' & the other end of s' is connected to a terminal T_2.
L: Soft iron core. It's so held that the coil can rotate freely without touching it. This makes the B linked with coil to be radial field. M: concave mirror attached to the phosphor bronze strip. This helps us to note the deflection of the coil using lamp & scale arrangement. The whole arrangement is enclosed in a non-metallic case to avoid disturbance due to air.

* S' (Hair spring) :- provides passage of current for the coil, keeps the coil in pos^n, generates the restoring torque on the twisted couple.

* phosphor-bronze strip: is used ∵ it has small restoring torque per unit twist. Due to it, the galvanometer is very sensitive.

THEORY :- length, $l = PQ = RS$; breadth, $b = QR = SP$

n – no. of turns, $A = l \times b$ = Area of each turn of the coil

$B \rightarrow$ strength of mag. field in which coil is suspended. $I \rightarrow$ Current passing thro' the coil.

As, $\tau = nIBA\sin\theta$ (for a rect. coil carrying current placed in B)

If the mag. field is radial i.e., plane of the coil is parallel to direct^n of B.

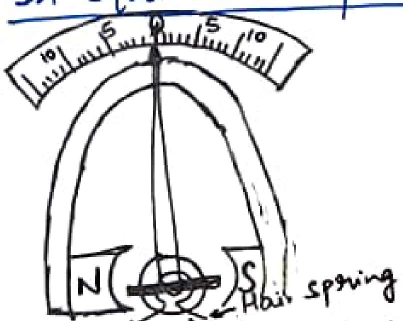
$\theta = 90° \Rightarrow \sin\theta = 1 \quad \therefore \tau = nIBA$

Due to this torque, the coil rotates. The phosphor bronze strip gets twisted. As a result of it, a restoring torque comes into play in the phosphor bronze strip, which would try to restore the coil back to original pos^n.

Let ϕ be the twist produced in the phosphor bronze strip due to rotation of the coil & k be the restoring torque per unit twist then,

$$\tau = k\phi$$

In equilibrium pos^n: Deflecting Torque = Restoring Torque



$$nIBA = k\phi$$

$$\Rightarrow I = \frac{k}{nBA}\phi$$

$$\Rightarrow \boxed{I = G\phi}$$

G → Galvanometer constant

Sensitivity of Galvanometer :-

(a) Current sensitivity :- Deflection produced in the galvanometer when a unit current flows through it.

$$I_s = \frac{\phi}{I} = \frac{nBA}{k} \qquad \text{Unit: rad } A^{-1} \text{ or div. } A^{-1}$$

(b) Voltage sensitivity :- Deflection produced in the galvanometer when a unit voltage is applied across two terminals of the galvanometer.

$$V_s = \frac{\phi}{V} = \frac{\phi}{IR} = \frac{nBA}{kR} = \frac{I_s}{R}$$; R: Resistance of the Galvanometer.

Cond^n for sensitive Galvanometer :- A galvanometer is said to be very sensitive if it shows large deflection even when a small current is passed thro' it.

As $\phi = \frac{nBA}{k} I$

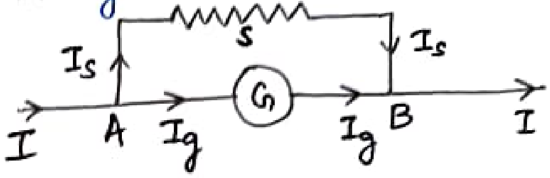
ϕ will be large if $\left(\frac{nBA}{k}\right)$ is large $\Rightarrow$ n: large, B: large, A: large & k: small

Galvanometer can't be used as an Ammeter :-

* It gives full scale deflection for current of μA range.
* For measuring, I ; G has to be connected in series, but it has large

R, which will change the value of I.

SHUNT:- It's a low resistance connected in parallel with the galvanometer or ammeter. It protects the galvanometer or ammeter from the strong currents.



Total current = I, Resist of Gal. = G,
Resist. of the shunt = S.

$$I = I_g + I_s$$

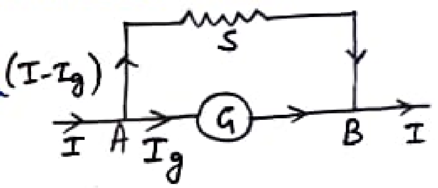
$$V_{AB} = I_g G = I_s S = (I - I_g) S$$

$$\Rightarrow I_g (G+S) = IS \Rightarrow \boxed{I_g = I\left(\frac{S}{G+S}\right)}$$

$$I_g G = I_s S \Rightarrow (I - I_s) G = I_s S \Rightarrow IG = I_s(G+S) \Rightarrow \boxed{I_s = I\left(\frac{G}{G+S}\right)}$$

Use of shunt: to protect the galvanometer from strong currents, for conversion of a galvanometer into an ammeter, used for increasing the range of Ammeter.

AMMETER:- It's a low R galvanometer, used to measure current in a circuit, G → Resist. of galvanometer



n → no. of scale divisions in the galvanometer

K → figure of merit or current for one scale deflection in the galvanometer

Current which produces full scale deflection, $I_g = nk$

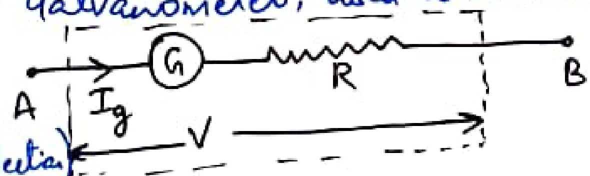
$$V_A - V_B = I_g G = (I - I_g) S \Rightarrow \boxed{S = \left(\frac{I_g}{I - I_g}\right) G}$$

The effective Resist, R_p of Ammeter (i.e., shunted galvanometer):

$$\frac{1}{R_p} = \frac{1}{G} + \frac{1}{S} = \frac{S+G}{GS} \Rightarrow \boxed{R_p = \frac{GS}{G+S}}$$

* S is low, R_p is very low & hence Ammeter has a much lower resist. than Galvanometer. An Ideal Ammeter has zero Resistance.

VOLTMETER: It's a high resist Galvanometer, used to measure the p.d. b/w any two pts of a circuit.



G → Resist. of galvanometer

As, $I_g = nk$ (for full scale deflection)

Total Resist. of Voltmeter = G+R

$$I_g = \frac{V}{G+R} \Rightarrow G+R = \frac{V}{I_g} \Rightarrow \boxed{R = \frac{V}{I_g} - G}$$

The eff. Resist, $R_s = G+R$

* A high Resist, R is connected in series with G ∴ Resist of voltmeter is very large as compared to that of G. The Resist of an ideal voltmeter is infinity.