

PHYSICS INDUCTION

An Institute of Science & Mathematics

Vinita Chawla

M.Sc. Physics (HRC, Delhi University)

M.Sc. Computer Science, B.Ed.,

CTET, HTET, PSTET, CSB Qualified

Contact: 91-8168258349

physicsinduction@gmail.com

Class XII : NOTES : Chapter-2 : Electrostatic Potential & Capacitance : PHYSICS

WORK-DONE IN BRINGING A TEST CHARGE (q) FROM ONE POINT TO ANOTHER IN THE ELECTRIC FIELD ($\vec{E}$) OF A GIVEN

CHARGE (Q): Let $Q > 0$, $q > 0$

↳ so small that it doesn't disturb the original configuration.

In bringing the charge, q , from posⁿ R to posⁿ P, we apply an external force, $\vec{F}_{ext}$ just enough to counter the repulsive Electric Force, $\vec{F}_E$

$$\text{i.e., } \vec{F}_{ext} = -\vec{F}_E$$

$R \rightarrow P$: infinitesimally slow const. spd; $a=0$. In this situation,

$$W_{ext} = -W_E$$

& gets fully stored in the form of P.E. of the charge, q_0 .

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$$W_{R \rightarrow P} = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = - \int_R^P \vec{F}_E \cdot d\vec{r}$$

At every point in electric field, a particle with charge, q_0 possesses certain electrostatic P.E., this W.D. increases its P.E. by an amount equal to P.E. diff. b/w pts R & P.

$$\text{Thus, P.E. diff., } \Delta U = U_P - U_R = W_{R \rightarrow P}$$

$$W_{ext} = U_P - U_R \Rightarrow W_E = -W_{ext} = U_R - U_P$$

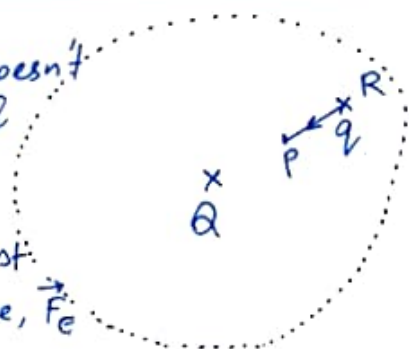
Note: 1) As $U_P - U_R$ depends only on the initial & final positions of q_0 , $\Rightarrow W_E$ depends only on i & f posⁿ. This is the fundamental charact of conservative force.

2) W.D. = ΔU , Actual value of P.E. isn't physically significant. ΔU is significant.

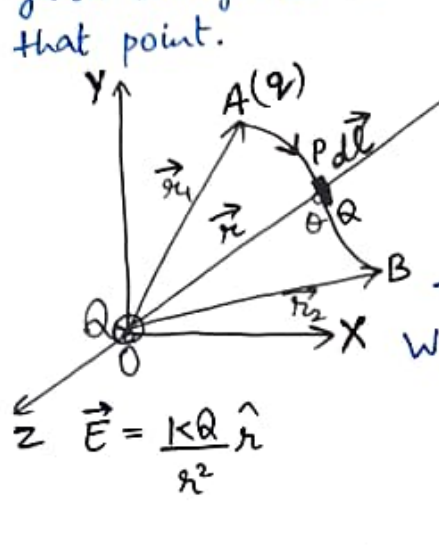
$\therefore$ we can add arb. const, α to P.E: $(U_P + \alpha) - (U_R + \alpha) = U_P - U_R$

$\Rightarrow$ We can choose a pt, where P.E. = 0

E.P. $E = 0$ (at ∞) [for the sake of our convenience] ①



Electrostatic P.E. :- P.E. of charge, q at a point in the electric field of given charge, Q is the W.D. in bringing the charge, q from infinity to that point.



W.D. in bringing the charge from P to Q:
 W.D. (P → Q): $dW = \vec{F} \cdot d\vec{l} = -qE(dr)\cos\theta = -qEdr$
 Total W.D. in bringing the charge from A to B:
 $W_{AB} = -q \int_A^B \vec{E} \cdot d\vec{r}$ (Same path b/w A & B)
 $= -q \int_{r_1}^{r_2} \vec{E} \cdot d\vec{r} = -KQq \int_{r_1}^{r_2} \frac{1}{r^2} dr = -KQq \left[-\frac{1}{r} \right]_{r_1}^{r_2}$
 $\Rightarrow W_{AB} = KQq \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$
 $\uparrow$
 V (Electrostatic P.E.)

ELECTROSTATIC POTENTIAL :- (V)

Electrostatic Potential, V at any point (P) in the electrostatic field of a given charge (Q) is defined as the W.D. in bringing a unit positive charge (without acceleration) from infinity to that point.

Electrostatic Pot. at a point in the field is defined as the negative of line-integral of electric field intensity from ∞ to that point.

$$V = - \int_{\infty}^P \vec{E} \cdot d\vec{r} = - \int_{\infty}^P \vec{E} \cdot d\vec{r} = \frac{W_{\infty P}}{q} = \frac{U}{q}$$

Higher Pot. $\rightarrow$ Lower Pot.

ELECTROSTATIC POTENTIAL DIFFERENCE :-

Electric Potential difference b/w two points A and B in an electrostatic field is defined as the amount of W.D. in carrying a unit positive test charge from A to B along any path b/w the two points.

$$V_{AB} = \frac{W_{AB}}{q} = - \int_A^B \vec{E} \cdot d\vec{r} = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{r} = -KQ \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

Unit of P.d. :- Volt, JC^{-1} , NmC^{-1}

1 Volt :- When 1J of Work is done in moving a +ve charge of 1C from one pt to the other, against the Electrostatic force of the field.

$$= KQ \left(\frac{1}{r_B} - \frac{1}{r_A} \right) = V_B - V_A$$

ELECTROSTATIC FORCES ARE CONSERVATIVE:-

F_E : conservative $\Rightarrow$ W.D. in moving a unit positive test charge over a closed path in an Electric Field is zero.

$$\frac{W_{AB}}{q} = - \int_A^B \vec{E} \cdot d\vec{r} = V_B - V_A \quad \text{--- (1)}$$

(along L)

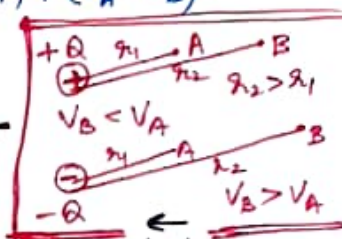
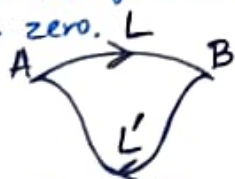
$$\frac{W_{BA}}{q} = - \int_B^A \vec{E} \cdot d\vec{r} = V_A - V_B \quad \text{--- (2)}$$

(along L')

From (1) & (2):

$$\frac{W_{ABA}}{q} = (V_B - V_A) + (V_A - V_B)$$

$$\Rightarrow \frac{W_{ABA}}{q} = 0$$



i) Electric Potential due to a point charge:-

Let $Q > 0$ To find:- Potential at point, P, $\vec{OP} = \vec{r}$
(W.D. in bringing a unit +ve test charge from ∞ to the pt. P)

At some intermediate posⁿ, A on the path,

Electrostatic force on a unit positive charge = $\frac{Q \times 1}{4\pi\epsilon_0 x^2} \hat{x}$

W.D. against this force from x' to $x' + \Delta x'$ is,

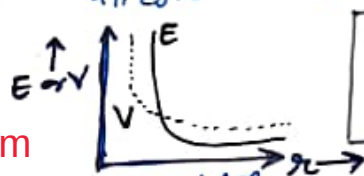
$$\Delta W = - \frac{Q}{4\pi\epsilon_0 x^2} \Delta x \quad (-ve \text{ sign } \because \Delta x < 0, \Delta W = +ve)$$

$\therefore$ Total W.D. (W) by external force:

$$W = - \int_{\infty}^r \frac{Q}{4\pi\epsilon_0 x^2} dx = \frac{-Q}{4\pi\epsilon_0} \left[-\frac{1}{x} \right]_{\infty}^r = \frac{Q}{4\pi\epsilon_0 r} = \frac{kQ}{r} \quad \left[\because k = \frac{1}{4\pi\epsilon_0} \right]$$

$$\Rightarrow V(r) = \frac{kQ}{r}$$

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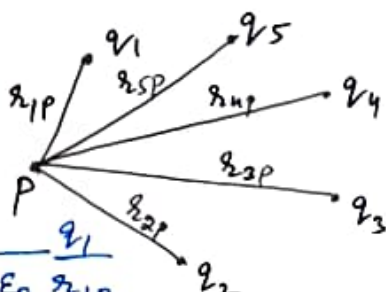


$$\begin{aligned} E &\propto \frac{1}{r^2} \\ V &\propto \frac{1}{r} \end{aligned}$$

Note:- for $Q < 0$, $V < 0 \Rightarrow W_{ext} = -ve \Rightarrow W_{electrostatic} = +ve$
 $\therefore$ for $Q < 0$, $F \rightarrow$ Attractive, F & disp. have the same directⁿ

(ii) Potential due to a system of charges:-

Consider a system of charges $q_1, q_2, q_3, \dots, q_n$ with position vectors $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$, relative to some origin.



The Potential, V_1 at P due to the charge, $q_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1p}}$
Illy " V_2 at P " " " " ; $q_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2p}}$

$$\& V_3 = \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_{3p}} \quad \text{Illy, } V_n = \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_{np}}$$

The Potential at P due to the total charge configuration, V is:
 $V = V_1 + V_2 + \dots + V_n$ [by Superposition Principle]

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \dots + \frac{q_n}{r_{np}} \right]$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{ip}} = k \sum_{i=1}^n \frac{q_i}{r_{ip}}$$

For continuous charge distribution :-

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = k \int \frac{dq}{r}$$



* If q is distributed uniformly along the length, L : $dq = \lambda dl$
 λ : Linear charge density (Cm^{-1})
 $\therefore V_L = k \int \frac{\lambda dl}{r}$

* If q is distributed uniformly over an area, S : $dq = \sigma ds$
 σ : Surface charge density (Cm^{-2})
 then, $V_S = k \int \frac{\sigma ds}{r}$

* If q is distributed uniformly over a volume, V : $dq = \rho dv$
 ρ : Volume charge density (Cm^{-3})
 then, $V_v = k \int \frac{\rho dv}{r}$

$\Rightarrow$ Net Potential at any point due to gp. of discrete pt charges as well as three types of continuous charge distribution can be written as:

$$V = k \left[\sum_{i=1}^n \frac{q_i}{r_i} + \int \frac{\lambda dl}{r} + \int \frac{\sigma ds}{r} + \int \frac{\rho dv}{r} \right]$$

(iii) Potential due to an Electric Dipole :-

Electrostatic Pot. follows the superposition Principle. Thus, potential due to a dipole is the sum of potentials due to charges q & $-q$.

From figure : $r^2 = x^2 + y^2$, $x = r \cos \theta$, $y = r \sin \theta$

$$r_1^2 = (x-a)^2 + y^2 = x^2 + y^2 - 2ax + a^2 = r^2 + a^2 - 2ar \cos \theta$$

$$\text{Ify, } r_2^2 = (x+a)^2 + y^2 = x^2 + y^2 + 2ax + a^2 = r^2 + a^2 + 2ar \cos \theta$$

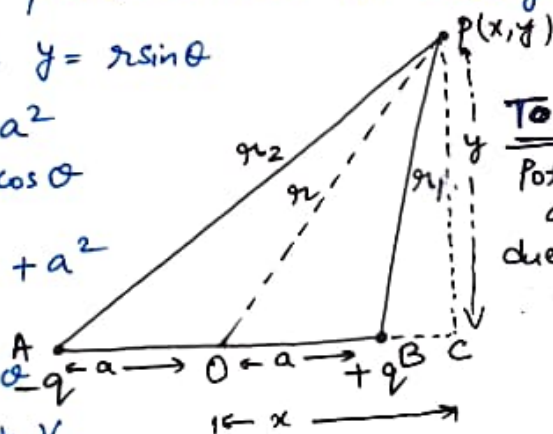
Electrostatic Pot. at $P = V = V_1 + V_2$

$$\Rightarrow V = \frac{kq}{r_1} - \frac{kq}{r_2}$$

$$[\text{where, } k = \frac{1}{4\pi\epsilon_0}]$$

$$= \frac{kq}{(r^2 - 2ar \cos \theta + a^2)^{1/2}} - \frac{kq}{(r^2 + 2ar \cos \theta + a^2)^{1/2}}$$

$$= \frac{kq}{r \left(1 - \frac{2a}{r} \cos \theta + \frac{a^2}{r^2} \right)^{1/2}} - \frac{kq}{r \left(1 + \frac{2a}{r} \cos \theta + \frac{a^2}{r^2} \right)^{1/2}}$$



To Find :-
 Potential at $P(x, y)$ due to dipole

for $r \gg a$, $\frac{a^2}{r^2}$ is negligible (retain terms only upto the 1st order $\frac{a}{r}$)

$$\therefore V = \frac{kq}{r} \left[\left(1 - \frac{2a \cos \theta}{r}\right)^{-1/2} - \left(1 + \frac{2a \cos \theta}{r}\right)^{-1/2} \right]$$

$$= \frac{kq}{r} \left[1 + \frac{a \cos \theta}{r} - 1 + \frac{a \cos \theta}{r} \right] \quad \left(\because (1 \pm x)^n = (1 \pm nx) \right)$$

$$= \frac{kq}{r} \times \frac{2a \cos \theta}{r} = \frac{kP \cos \theta}{r^2}$$

At large distance, $r \gg a$,
V falls off as $1/r^2$, not as $1/r$

Thus, $V = \frac{kP \cos \theta}{r^2} = \frac{k \vec{P} \cdot \hat{r}}{r^2} \quad (r \gg a)$

V depends on r as well as $\angle$ b/w $\vec{r}$ & $\vec{P}$

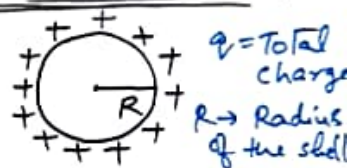
On axial line of Electric Dipole: $\theta = 0^\circ, \pi$ $\begin{pmatrix} \cos 0^\circ = 1 \\ \cos \pi = -1 \end{pmatrix}$

$$\therefore V = \pm \frac{kP}{r^2}; \quad V \propto \frac{1}{r^2} \quad \text{www.physicsinduction.com}$$

On Equatorial line of dipole: $\theta = 90^\circ, \cos 90^\circ = 0 \Rightarrow V = 0$

iv) Electrostatic Potential due to Uniformly charged spherical shell:-

• For a Uniformly charged spherical shell, the Electric Field outside is as if the entire charge is concentrated at the centre.



Thus, Pot. outside the shell: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (r \geq R)$

• The Electric Field inside the shell is zero: $V = \text{constant} \quad (r \leq R)$
 $\Rightarrow$ Pot is constant inside the shell: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$

EQUIPOTENTIAL SURFACES:- An equipotential surface is that at every point of which electric potential is the same.

(i) Pot diff b/w 2 pts A & B ($V_B - V_A$) = W.D. in carrying a unit +ve test charge from A to B (W_{AB})

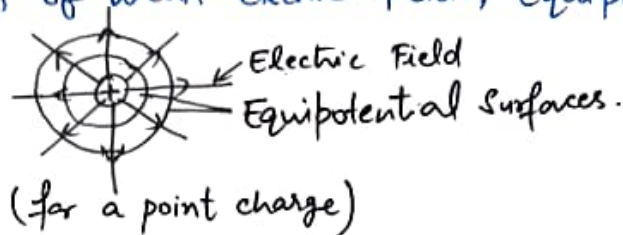
if $V_A = V_B \Rightarrow W_{AB} = 0 \Rightarrow$ No Work is done in moving the test charge from one pt. of equipotential surface to the other.

(ii) As $dW = 0$
 $\Rightarrow \vec{E} \cdot d\vec{r} = 0$

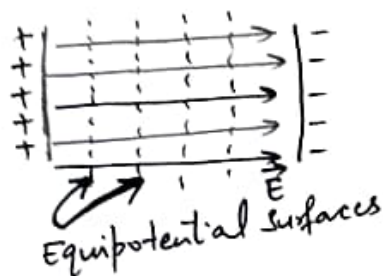
$$\Rightarrow E dr \cos \theta = 0 \Rightarrow \theta = 90^\circ \Rightarrow \vec{E} \perp d\vec{r}$$

$\Rightarrow$ Equipotential surfaces are always $\perp$ to the field lines.

(iii) In the region of strong electric field, Equipotential surfaces are close together and in the region of weak electric fields, Equipotential surfaces are far apart.



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RELATION BETWEEN FIELD AND POTENTIAL :- $AB = dr$

A & B are so close that $\vec{E}$ b/w A & B is uniform.

P.d. (dV) b/w B & A = W.D. in moving unit positive test charge from A to B.

$$dV = \text{Force on unit charge} \times AB$$

$$= \vec{E} \cdot d\vec{r} = E dr \cos 180^\circ = -E dr \quad (\text{-ve sign indicates that work is done against the } \vec{E})$$

$$\therefore \boxed{E = -\frac{dV}{dr}}$$

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- Electric Field is in the directⁿ in which the potential decreases steepest.
- Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at that pt.

ELECTROSTATIC POTENTIAL ENERGY :- The Electric P.E. is the energy possessed by a system of point charges by virtue of their positions.

We may define, Electric P.E. of a system of point charges as the total amount of W.D. in bringing the various charges to their respective positions from infinitely large mutual separations.

POTENTIAL ENERGY OF A SYSTEM OF CHARGES :- = W.D. in assembling all the charges at a given location

Let's consider all the charges at infinity initially & determine the W.D. by an external agency to bring the charges at the given locations.

Suppose, first the charge, q_1 is brought from infinity to the point, $\vec{r}_1$.

W.D. in bringing q_1 from infinity to r_1 , $W_1 = 0$ { $\because$ no ext. field against which work needs to be done }

Electric Pot. due to q_1 , $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1p}}$ { $r_{1p} \rightarrow$ dist. of pt, p from the location of q_1 }

q_1

(6)

W.D. in bringing charge, q_2 from infinity to the point, $\vec{r}_2$: W_2

$$W_2 = (\text{Potential due to } q_1) \cdot q_2$$

$$= \frac{K q_1 q_2}{r_{12}}$$

[r_{12} : distance b/w pts 1 & 2]

①
1

②
2

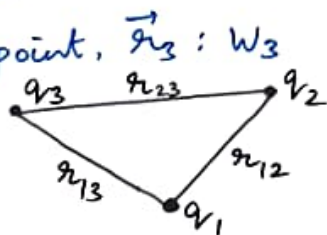
If $q_1 q_2 > 0$: P.E. - +ve, F_{elec} - Repulsive, W.D. - +ve (∞ to finite dist.)

If $q_1 q_2 < 0$: P.E. - -ve, F_{elec} - attractive, W.D. - -ve.

W.D. in bringing charge, q_3 from infinity to the point, $\vec{r}_3$: W_3

$$W_3 = (\text{Pot. due to } q_1) q_3 + (\text{Pot. due to } q_2) q_3$$

$$= \frac{K q_1 q_3}{r_{13}} + \frac{K q_2 q_3}{r_{23}}$$



$$\therefore \text{Total W.D.} = U = 0 + \frac{K q_1 q_2}{r_{12}} + \left(\frac{K q_1 q_3}{r_{13}} + \frac{K q_2 q_3}{r_{23}} \right)$$

For N point charges :- $U = K \sum_{\text{all pairs}} \frac{q_j q_k}{r_{jk}}$

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$$\Rightarrow U = \frac{1}{2} \sum_{j=1}^N \sum_{\substack{k=1 \\ j \neq k}}^N \frac{K q_j q_k}{r_{jk}}$$

{ ∵ in the summation set up, we count each pair twice }

Note :- The P.E. is characteristic of the present state of configuration, and not the way, the state is achieved.

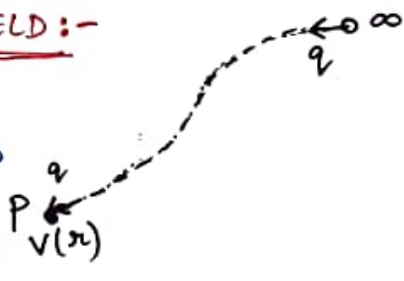
POTENTIAL ENERGY IN AN EXTERNAL FIELD :-

(i) Potential Energy of a single charge :-

Let V be the potential at a point, P due to an external field;

W.D. in bringing a charge, q from infinity to the pt., P:

$$W = \int_{\infty}^r \vec{F} \cdot d\vec{r} = q \int_{\infty}^r \vec{E} \cdot d\vec{r}$$



$$\therefore \text{E.P.E. of the charge, } q(\text{at } r); U(r) = -W.D. = -q \int_{\infty}^r \vec{E} \cdot d\vec{r} = \underline{qV(r)}$$

1eV: If an e^- is accelerated by the p.d. of $\Delta V = 1 \text{ volt}$.

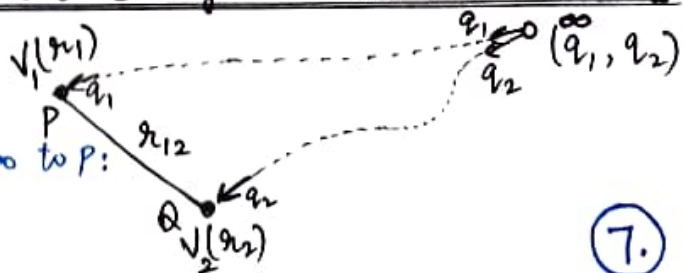
$$\text{E.P.E.} = qV = e\Delta V = (1.6 \times 10^{-19}) \times 1 = \underline{1.6 \times 10^{-19} \text{ J}}$$

This unit of energy is defined as 1eV. [1eV = $1.6 \times 10^{-19} \text{ J}$]

1 keV = 10^3 eV , 1 MeV = 10^6 eV , 1 GeV = 10^9 eV , 1 TeV = 10^{12} eV .

(ii) Potential Energy of a system of two charges in an Ext. Field :-

Let V_1 & V_2 be the potentials at points P & Q in an external field.



(i) W.D. in bringing the charge, q_1 from ∞ to P:

$$W_1 = q_1 V_1$$

⑦

(ii) Total Potential at Q is due to $\left[\begin{array}{l} \rightarrow \text{External Field. } (V_2) \\ \rightarrow \text{the charge, } q_1 \text{ at } P. \left(\frac{Kq_1}{r_{12}} \right) \end{array} \right]$

$\therefore$ W.D. in bringing the charge, q_2 from ∞ to Q;

$W_2 =$ W.D. against the ext. field + W.D. against the field due to q_1 ,

$$\Rightarrow W_2 = q_2 V_2 + \frac{Kq_1 q_2}{r_{12}} \quad [r_{12} = \text{dist. b/w } q_1 \text{ \& } q_2]$$

Thus, E.P.E. of the system = Total W.D. in assembling the configuration.

$$U = W = W_1 + W_2 = q_1 V_1 + q_2 V_2 + \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r_{12}} //$$

(iii) Potential Energy of a dipole in an External Field:-

In a uniform Electric Field, $\vec{E}$:

$$\vec{F}_{\text{net}} = 0 \quad (\because \vec{F}_1 = -\vec{F}_2)$$

but, torque, $\tau \neq 0$

$$\tau = F(AC) = qE(2a \sin \theta) = (q \times 2a) E \sin \theta$$

$$= PE \sin \theta = \vec{P} \times \vec{E}$$

The amt of W.D. in rotating the dipole by the external torque from θ_1 to $\theta_2 = W = \int_{\theta_1}^{\theta_2} \tau d\theta$

$$\Rightarrow W = \int_{\theta_1}^{\theta_2} PE \sin \theta d\theta = -PE(\cos \theta)_{\theta_1}^{\theta_2} = -PE(\cos \theta_2 - \cos \theta_1)$$

At $\theta_1 = \pi/2$, in this case, the W.D. against the external field, $\vec{E}$ in bringing $+q$ & $-q$ are equal & opposite and cancel out.

$$\text{i.e., } q[V_1 - V_2] = 0$$

$$\& \text{ Also, } W = -pE(\cos \theta - \cos 90^\circ) = -pE \cos \theta = -\vec{P} \cdot \vec{E}$$

This work is stored as P.E. of the system. $\therefore U = -\vec{P} \cdot \vec{E}$

Alternatively E.P.E., $U(\theta) = q[V_1 - V_2] - \frac{q^2}{(4\pi\epsilon_0)2a}$ { where V_1 : elec. pt on $+q$ at r_1
 V_2 : Elect. pt by ext field on $-q$ at r_2 }

But $V_1 - V_2 = -E(2a \cos \theta)$ [$\because$ displacement parallel to the force is $2a \cos \theta$]

$$\therefore U(\theta) = -q(2a)E \cos \theta - \frac{q^2}{(4\pi\epsilon_0) \times 2a} = -\vec{P} \cdot \vec{E} - \frac{q^2}{(4\pi\epsilon_0) \times 2a}$$

ELECTROSTATICS OF CONDUCTORS:-

Conductors - Mobile charge carriers

In Metallic conductors - Mobile charge carriers are electrons (affected by $\vec{E}$)

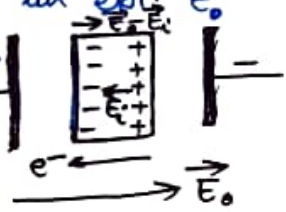
In Electrolytic conductors - charge carriers are both positive & negative ions.

(affected by $\vec{F}_E$ and chemical forces)

Results regarding Electrostatics of Conductors:- BEHAVIOUR OF CONDUCTORS IN ELECTROSTATIC FIELDS

(i) Inside a conductor, $\vec{E} = 0$: In static situation, the free charges have so distributed themselves that the electric field is zero, everywhere inside.

Consider a situation, where a conductor is held in an ext. $\vec{E}_0$. As a result, charges will be induced inside the conductor. They produce an induced $\vec{E}_i$, which opposes the flow of e^- . The flow of e^- stops when $\vec{E}_0 = \vec{E}_i$.



As applied & induced electric field inside the cond. are eq. & opp. $\therefore$ Net Elect. field in the interior of cond. is zero.

(ii) The interior of a conductor can have no excess charge in the static situation:-

As, Net Electric field in the interior of a cond., $\vec{E} = 0$

$$\therefore \oint \vec{E} \cdot d\vec{S} = 0 \Rightarrow \frac{Q}{\epsilon_0} = 0 \Rightarrow Q = 0 \because E_0 \neq 0$$

(iii) Excess charge resides on the outer surface of a conductor:- $\because$ $\nabla \cdot \vec{E} = \rho$ no net Q inside the conductor.

(iv) At the surface of a charged conductor, Electrostatic field must be normal to the surface at every point:- In static situation, charges of a cond. are rearranged & flow of charges stops. Therefore, comp. of $\vec{E}$ along the tangent to the surface of a conductor must be zero.

i.e., $E \cos \theta = 0$ (θ : $\angle$ b/w $\vec{E}$ & tangent to the surface)

$$\Rightarrow \cos \theta = 0 (\because E \neq 0)$$

$$\Rightarrow \boxed{\theta = 90^\circ}$$

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(v) Electrostatic Potential is constant throughout the volume of the conductor & has the same value (as inside) on its surface:-

As $E = 0$ (inside the conductor)

$$\Rightarrow -\frac{dV}{dr} = 0 \Rightarrow V = \text{constant (inside the cond.)}$$

E has no tangential comp. on the surface, no work is done in moving a small test charge, within the conductor & on its surface. i.e., $\nabla \cdot \vec{E} = \rho$ no p.d. b/w any two points inside or on the surface of the conductor.

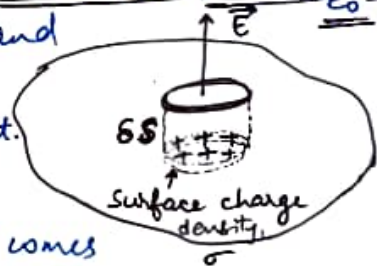
(vi) Electric Field at the surface of a charged conductor, $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$.

Choose a pill box (a short cylinder), partly inside and partly outside the surface of the conductor. It has a small area of cross-section, SS & negligible height.

$\vec{E} = 0$ (just inside), field is normal (just outside)

The contribution to the total flux thro' the pill box comes only from the outside (circular cross-section) of the pill box.

$$\phi = \pm E SS \quad [+ve \text{ for } \sigma > 0, -ve \text{ for } \sigma < 0]$$



$$E_{SS} = \frac{|\sigma| SS}{\epsilon_0} \Rightarrow \boxed{E = \frac{|\sigma|}{\epsilon_0}} \quad \text{If } \sigma > 0, \text{ Field is normal outward.}$$

$$\sigma < 0, \text{ Field is normal inward.}$$

(vii) Surface charge density is different at different points:

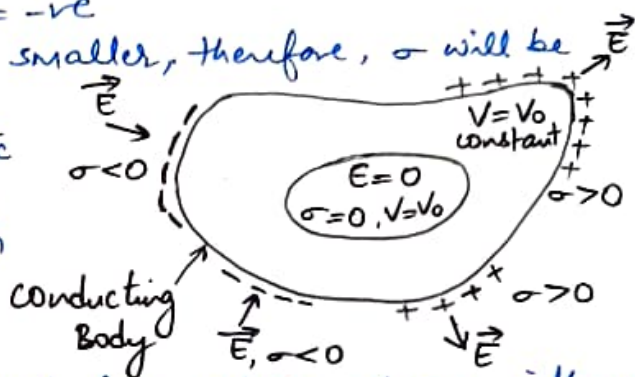
$\sigma = +ve$ if $q = +ve$; $\sigma = -ve$ if $q = -ve$

If radius of curvature is small, A is smaller, therefore, σ will be higher & vice-versa.

(viii) Electrostatic shielding: - Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence, field inside the cavity is always zero.

$\therefore$ to protect delicate instruments from external electric fields, we enclose them in hollow conductors. Such hollow conductors are called **Faraday Cages**. Ex: during lightning, safer to sit inside the car. A high voltage generated is enclosed in such a cage, which is earthed.

(ix) Action of points: - Conductors having sharp pts. σ & E : high on sharp pts.



DIELECTRICS AND POLARISATION: - www.physicsinduction.com

Dielectrics: - Those substances which transmit electric effects without actually conducting are called dielectrics. (affected by $\vec{E}$). Insulators are also dielectrics.

Dielectric strength: - Maximum $\vec{E}$ that a pure material can withstand under ideal conditions without breaking down. (i.e., without experiencing failure of its insulating properties), OR Maximum Voltage that can be applied to a given material, without causing it to breakdown.

In case of dielectrics, (V or kV per unit of thickness)

induced Electric field doesn't cancel the external Electric field. It only reduces it.

The extent of the effect depends upon the nature of the dielectrics.

Charge distribution of dielectric at the molecular level: -

Non-Polar Dielectrics: - In a Non-Polar molecule, the centres of +ve & -ve charges coincide. The molecule has no permanent (or intrinsic) dipole moment. $P=0$, symmetrical in shape.

For ex: $H_2, N_2, O_2, CO_2, C_6H_6, CH_4, CCl_4$

When a non-polar molecule is placed in an electric field, the centres of +ve & -ve charges get displaced & the molecule is then said to have been polarized. The induced dipole moment is prop. to the applied $\vec{E}$ & it is almost independent of temperature. Further, the induced dipole is parallel to the electric field right at the time of its creation.



Polar Dielectrics:- In a polar molecule, the centres of +ve & -ve charges are separated. Such molecules have permanent dipole moment, asymmetrical in shape. $p \neq 0$. For Ex: $HCl, H_2O, N_2O, NH_3, H_2S, C_2H_5OH, SO_2$ etc.

In the absence of $\vec{E}$, $\vec{P}$ of these polar molecules pt. in random directⁿ & cancel each other. Therefore, even though, each molecule has a dipole moment, the avg. moment per unit Volume is zero. On the application of $\vec{E}$, the dipole moments of these molecules align themselves parallel to the directⁿ of $\vec{E}$. But, this alignment is incomplete due to the thermal vibrations of molecules. Alignment of the molecules with the applied field increases if:

- (i) Electric intensity of the field is increased, (ii) Temp. is decreased.

Note:- The displacement of charges stops, when the ext. force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule)

Polarisation:- It's the phenom in which an alignment of +ve & -ve charges takes place within the dielectric resulting in no net increase in the charge of the dielectric.

Induced Dipole Moment, $P = \alpha E_0 E_0$  order of $\uparrow$ At. Volume

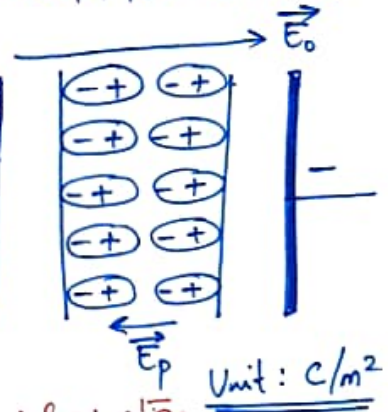
α : Atomic (molecular polarizability (m^3)), Most Atoms $\rightarrow \alpha: 10^{-29}$ to $10^{-20} m^3$

Units: $E_0 \rightarrow C^2 N^{-1} M^{-2}$, $E_0 \rightarrow NC^{-1}$, $\alpha \rightarrow m^3$, $P \rightarrow Cm$

Dipole Moment Density:-

Consider a non-polar dielectric slab placed in an electric field, E_0 maintained b/w the two plates. Suppose, all its atoms are uniformly polarised in the directⁿ of E_0 & x is the displacement b/w $\pm$ qs.

Dipole Moment of each atom, $P = qx$
No. density, $N = \frac{\text{no. of atoms}}{\text{Volume}}$
 $\therefore$ Dipole Moment Density,
 $P = P/v = Np = Nqx$



$\therefore$ Effective Electric Field in a polarized dielectric is $E = E_0 - E_p$ Dipole Moment Density / Electric Polarization E : Reduced value of the electric field.

$E_p = \frac{\sigma_x}{\epsilon_0} = \frac{1}{\epsilon_0} \left(\frac{Qi}{A} \right) = \frac{P}{\epsilon_0}$ www.physicsinduction.com

Electric Susceptibility:- Electric Polarization, $P \propto$ Reduced Value of Electric field, E

$P = \epsilon_0 \chi E$; $\chi \rightarrow$ electric susceptibility of the dielectric. It's dimensionless.

The electric susceptibility describes the electrical behaviour of the dielectric. It has different values for different dielectrics.

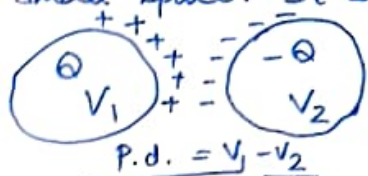
$\chi = 0$ (for vacuum)

$E = E_0 - \frac{P}{\epsilon_0} = E_0 - \frac{\epsilon_0 \chi E}{\epsilon_0} = E_0 - \chi E$

$E_0 = E(1 + \chi)$

$\Rightarrow \frac{E_0}{E} = 1 + \chi \Rightarrow \boxed{K = 1 + \chi}$ $\leftarrow$ Relⁿ b/w dielectric constant & electrical susceptibility of the material.

CAPACITANCE: A capacitor was formerly called a condenser, is an arrangement for storing a large amount of electric charge in a small space. It is a system of two conductors separated by an insulator. Even a single conductor can be used as a capacitor by assuming the other at infinity.



Electric Field in the region b/w the conductors, $E \propto Q$ (Coulomb's law)
P.D. is also prop. to Q . Because, if we go on giving charge to a conductor its potential keeps on rising.

Charge, $Q \propto$ Potential, V

$$\Rightarrow Q = CV \Rightarrow \boxed{C = Q/V}$$

C : Capacitance of the conductor

Unit of capacitance: Farad, F (S.I.) ; esu of capacitance/statfarad (esu)

1 Farad:- Capacitance of a conductor is said to be $1F$ if its potential rises through $1V$ when a charge of $1C$ is given to it.

$$\text{If } Q = 1C, V = 1V \Rightarrow C = \frac{1C}{1V} = 1CV^{-1} = \underline{1F}$$

$$\text{Also, } \underline{1F} = \frac{1C}{1V} = \frac{3 \times 10^9 \text{ statCoulomb}}{(1/300) \text{ statvolt}} = \underline{9 \times 10^{11} \text{ statfarad}}$$

Dim. Formula:- $C = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$

SYMBOL

fixed C



Variable C

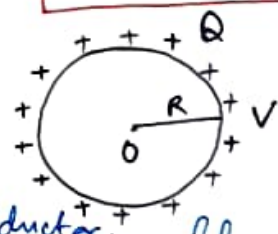


Capacitance of an isolated spherical conductor:-

Let Q : charge given to the spherical conductor.

V : Potential on the surface of the spherical conductor.

The charge spreads uniformly over the outer surface of the sphere, whether, the sphere is hollow or solid.



$\therefore$ Potential at every pt. on the surface of the sphere is same. As, the sphere behaves as if the entire charge were concentrated at the centre of the sphere, therefore, potential at any point on the surface of the sphere in free space, $V = \frac{Q}{4\pi\epsilon_0 R}$

$$\text{As, } C = \frac{Q}{V} \quad \text{www.physicsinduction.com}$$

$$\therefore \boxed{C = 4\pi\epsilon_0 R}$$

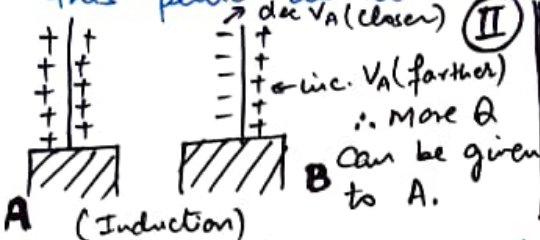
C of Earth:- Taking Earth to be a sphere of $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

$$C = \frac{6.4 \times 10^6}{9 \times 10^9} = 0.711 \times 10^{-3} F = \underline{711 \mu F} \leftarrow \text{very large } \therefore \underline{\text{Unlimited } Q}$$

PRINCIPLE OF CAPACITOR:- Consider an insulated metal plate, A

Let same positive charge be given to this plate, till its pot.

becomes maximum. No further charge can be given to this plate as it would leak out.

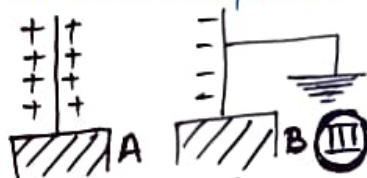


Consider another insulated metal plate B held near plate A. By induction, $-Q$ is produced on the nearer face of B & an equal $+Q$ develops on the farther face of B.

Induced $-Q$ on B tends to dec. the pot. of A & induced $+Q$ tends to inc. the pot. of A. Induced $-Q$ is closer $\therefore$ more effective.

$\therefore$ Overall V_A reduces & more charge can be given to A to raise its pot. to max.

Connect the plate B to earth. due to this V_A is greatly reduced. Thus, a large amt. of charge can be given to A to raise its pot. to max.



★ The capacitance of an insulated charged conductor is increased appreciably by bringing near it an earth-connected uncharged conductor.

★ Capacitor with large capacitance can hold large amount of charge Q at a relatively small V .

★ There is a limit to the amt of charge that can be stored on a given capacitor without significant leaking.

THE VALUE OF C DEPENDS UPON: www.physicsinduction.com

(i) The size & shape of the conductor, (ii) the nature of the medium surrounding the conductor & (iii) the posⁿ of the neighbouring charges.

THE PARALLEL PLATE CAPACITOR:-

Let A be the area of each plate ($d^2 \ll A$) & d " " distance b/w the plates

In the outer regions I & II

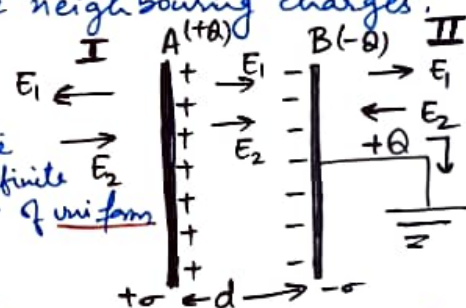
$$\text{net } E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

In the inner region b/w the plates: $E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$$AB, V = Ed \therefore V = \frac{Qd}{A\epsilon_0}$$

$$\Rightarrow C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

★ For plates with finite A , σ will not be strictly uniform. However for $d^2 \ll A$, this can be ignored.



$$\text{for } A = 1\text{m}^2, d = 1\text{mm}: C = \frac{8.85 \times 10^{-12} \times 1}{10^{-3}} = 8.85 \times 10^{-9} \text{ F}$$

$$\text{for } C = 1\text{F}, d = 1\text{mm}: A = \frac{Cd}{\epsilon_0} = \frac{1 \times 10^{-3}}{8.85 \times 10^{-12}} = 0.11 \times 10^9 \text{ m}^2 \approx 10^9 \text{ m}^2$$

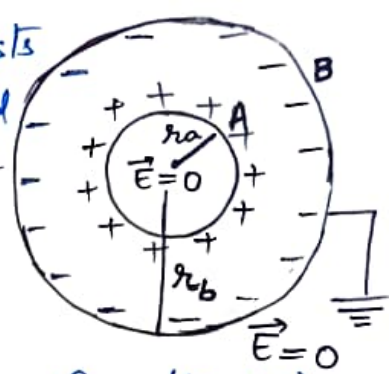
Effect of Dielectric on Capacitance:-

$$C_m = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d} = \epsilon_0 C_0 = k C_0 \Rightarrow k = \frac{C_m}{C_0}$$

When an insulating material is introduced in the space b/w the charged plates of a capacitor, its capacitance increases.

for plate 30km in length & breadth (How big 1F is...)

SPHERICAL CAPACITOR A spherical capacitor consists of a hollow conducting sphere A of radius, r_a , surrounded by another concentric conducting spherical shell B of radius, r_b . $E=0$ for $r < r_a$ & $r > r_b$.



Potential of inner sphere A is, $V_A = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$

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$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b} \right) = \frac{Q}{4\pi\epsilon_0} \frac{(r_b - r_a)}{r_a r_b}$$

As, outer sphere B is earthed, therefore, its pot; $V_B = 0$

P.d. b/w the two spheres A & B = $V_A - V_B = \frac{Q}{4\pi\epsilon_0} \frac{(r_b - r_a)}{r_a r_b}$

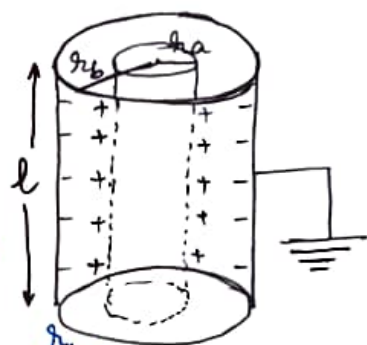
$$\text{As, } C = \frac{Q}{V} = \frac{Q}{V_A - V_B} = \frac{4\pi\epsilon_0 r_a r_b}{(r_b - r_a)}$$

CYLINDRICAL CAPACITOR:- $\lambda = \frac{Q}{l}$

Electric field Intensity in the region b/w the two cylinders, $E = \frac{\lambda}{2\pi\epsilon_0 r}$ where, $r_a \leq r < r_b$

The potential diff. b/w the two cylinders, i.e.,

$$V_B - V_A = - \int_{r_a}^{r_b} E \cdot dr = - \frac{\lambda}{2\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r} = - \frac{\lambda}{2\pi\epsilon_0} (\ln r)_{r_a}^{r_b} = \frac{-\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$



$$\therefore V_A - V_B = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$

↑
+ve, because

inner cylinder is at higher potential

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$$\therefore C = \frac{Q}{V} = \frac{Q}{V_A - V_B} = \frac{\lambda l}{\left(\frac{\lambda}{2\pi\epsilon_0} \right) \ln \left(\frac{r_b}{r_a} \right)} = \frac{2\pi\epsilon_0 l}{\ln \left(\frac{r_b}{r_a} \right)}$$

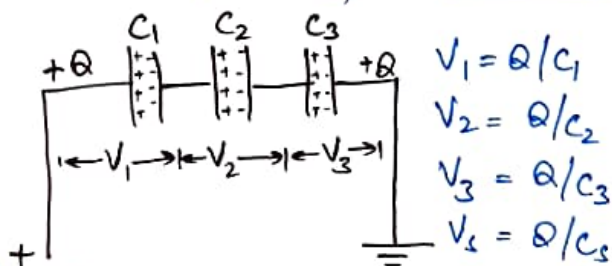
COMBINATION OF CAPACITORS:-

(a) Capacitors in series:- A no. of capacitors are said to be connected in series, if each capacitor acquires the same charge & sum of the voltage drops across all the capacitors is equal to the voltage (p.d.) across its terminals.

$$\text{As, } V = V_1 + V_2 + V_3$$

$$\Rightarrow \frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \Rightarrow \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\text{for } n \text{ capacitors, } \frac{1}{C_s} = \sum_{i=1}^n \frac{1}{C_i}$$



$$V_1 = Q/C_1$$

$$V_2 = Q/C_2$$

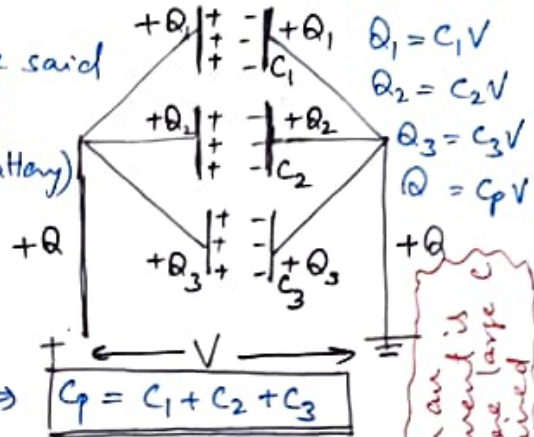
$$V_3 = Q/C_3$$

$$V_s = Q/C_s$$

Such an arrangement is used where the working voltage is high & a single capacitor can't sustain it

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

(b) Capacitors in parallel:- The capacitors are said to be connected in parallel if potential diff across each capacitor is the same ($= V$ of the battery) & the total charge is equal to the sum of the charges on the individual capacitors.



$$\text{As, } Q = Q_1 + Q_2 + Q_3$$

$$\Rightarrow C_p V = C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3) V \Rightarrow \boxed{C_p = C_1 + C_2 + C_3}$$

for n capacitors, $C_p = \sum_{i=1}^n C_i$

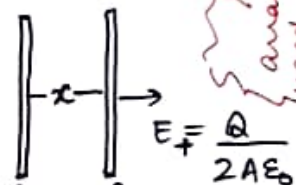
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FORCE BETWEEN THE PLATES OF A CAPACITOR:-

Consider a parallel-plate capacitor with plate area A .

Suppose, a positive charge, $+Q$ is given to one plate & a negative charge, $-Q$ to the other plate. The Electric field due to only the +ve plate is, $E_+ = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$ at all pts, if the plate is large. The -ve charge, $-Q$ finds itself in the field of this +ve charge. The force on $-Q$ is, therefore, $\underline{F} = -QE_+ = (-Q) \frac{Q}{2A\epsilon_0} = \underline{\underline{-\frac{Q^2}{2A\epsilon_0}}}$

$$\therefore \text{Magnitude of force, } F = \frac{Q^2}{2A\epsilon_0}$$



Such an arrangement is used, where large C is required.

ENERGY STORED IN A CAPACITOR:-

In transferring +ve charge from conductor II to conductor I, work will be done externally.

Consider an intermediate situation when the conductors I & II have charges Q' & $-Q'$ resp.

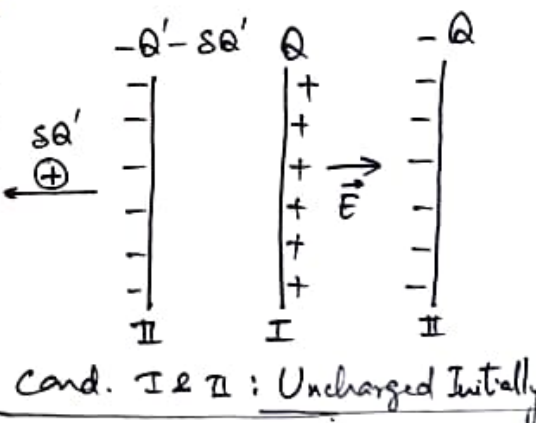
$$V'(\text{b/w I \& II}) = Q'/C$$

W.D. in transferring a small charge, $\delta Q'$ from cond. II to cond I,

$$\delta W = V' \delta Q' = \frac{Q'}{C} \delta Q'$$

$$\therefore \text{Total W.D.} = W = \int_0^Q \frac{Q'}{C} \delta Q' = \frac{1}{C} \left[\frac{Q'^2}{2} \right]_0^Q = \frac{Q^2}{2C}$$

$$\therefore W = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$



Cond. I & II: Uncharged Initially

As, $F_E \rightarrow$ conservative $\therefore W \rightarrow$ P.E. $\therefore$

$$\boxed{U = W = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV}$$

Energy stored in a parallel plate capacitor:-

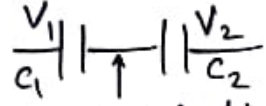
$$U = \frac{Q^2}{2C} = \frac{(A\sigma)^2}{2 \epsilon_0 A} \cdot \frac{d}{A} = \frac{A^2 (\epsilon_0 E)^2}{2} \cdot \frac{d}{\epsilon_0 A} \quad (\because E = \frac{\sigma}{\epsilon_0})$$

$$\Rightarrow U = \frac{1}{2} \epsilon_0 E^2 \times Ad$$

$$\therefore \text{Energy Density, } u = \frac{U}{V} = \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

$\leftarrow$ It holds true for electric field due to any configuration of charges.

COMMON POTENTIAL :- When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher pot. to the other at lower pot. This flow continues till their potentials become equal.



Total charge before sharing = $Q = C_1 V_1 + C_2 V_2$

Let V be the common potential on sharing charges.

Total charge after sharing = $Q = (C_1 + C_2) V$

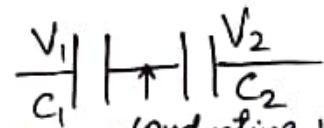
As, no charge is lost in the process of sharing, therefore

$$(C_1 + C_2) V = C_1 V_1 + C_2 V_2$$

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$$\Rightarrow V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

LOSS OF ENERGY ON SHARING CHARGES :-



Total Energy before sharing charges, $E_1 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$

Total Energy after sharing charges, $E_2 = \frac{1}{2} (C_1 + C_2) V^2$

$$\Rightarrow E_2 = \frac{1}{2} (C_1 + C_2) \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right]^2 \quad \left[\because V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right]$$

$$\Rightarrow E_2 = \frac{(C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

$$\therefore E_1 - E_2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{(C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{C_1 V_1^2 (C_1 + C_2) + C_2 V_2^2 (C_1 + C_2) - (C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{C_1 C_2 (V_1^2 + V_2^2 - 2V_1 V_2)}{2(C_1 + C_2)}$$

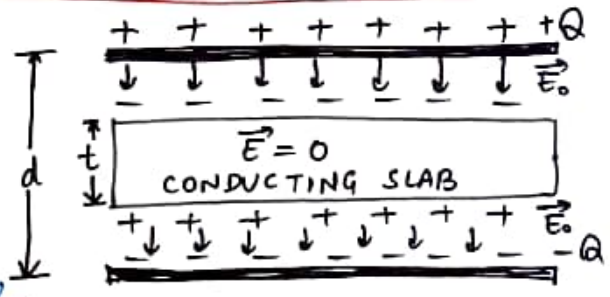
$$\Rightarrow E_1 - E_2 = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)} \rightarrow +ve \therefore E_1 - E_2 > 0 \Rightarrow E_1 > E_2$$

Loss of energy appears in the form of sparking & heat produced.

PARALLEL PLATE CAPACITOR WITH A CONDUCTING SLAB :-

The capacitance of a parallel plate capacitor of area, A & plate separation, d with vacuum/air in b/w is: $C_0 = \frac{\epsilon_0 A}{d}$

when a conducting slab of area, A & thickness $t < d$ is introduced b/w the plates, the free charges in the conductor flow. The charges $\pm Q$ appear on the two faces of the slab (fig) reduces the $\vec{E}$ in the interior of the slab to zero. The original



uniform electric field, E_0 now exists over a distance $(d-t)$.

$$\therefore \text{p.d. b/w the plates, } V = E_0(d-t) = \frac{\sigma}{\epsilon_0}(d-t) = \frac{Q}{A\epsilon_0}(d-t)$$

$$\text{As, } C = \frac{Q}{V} \therefore C = \frac{Q(A\epsilon_0)}{Q(d-t)} = \frac{\epsilon_0 A}{d-t} = \frac{\epsilon_0 A/d}{(1-t/d)} = \frac{C_0}{(1-t/d)}$$

$$\Rightarrow \boxed{C = \frac{C_0}{1-t/d}} \quad \text{clearly, } \underline{C > C_0}$$

PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB:-

$$C_0(\text{vacuum}) = \frac{\epsilon_0 A}{d}$$

when a dielectric slab is introduced b/w the plates, the molecules in the slab get polarized in the directⁿ of E_0 . The electric polarization vector, P in the directⁿ of E_0 induces an electric field E_p opposite to E_0 .

$\therefore$ Effective field inside the dielectric is $E = E_0 - E_p$

Outside the dielectric, field remains E_0 only.

$\therefore$ p.d. b/w the two plates, $V = E_0(d-t) + Et$

$$\Rightarrow V = E_0(d-t) + \frac{E_0 t}{K} \quad \left[\frac{E_0}{E} = \epsilon_r \text{ or } K \right]$$

$$\therefore V = E_0(d-t) + \frac{E_0 t}{K}$$

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$$\Rightarrow V = E_0(d-t + \frac{t}{K}) = \frac{\sigma}{\epsilon_0}(d-t + \frac{t}{K}) = \frac{Q}{A\epsilon_0}(d-t + \frac{t}{K})$$

$$\therefore \text{Capacitance, } \underline{C = \frac{Q}{V} = \frac{\epsilon_0 A}{d-t + \frac{t}{K}} = \frac{\epsilon_0 A}{d-t(1 - \frac{1}{K})}} \quad , \quad \underline{C > C_0}$$

Magnitude of the Induced Charge:-

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}, \quad E = \frac{E_0}{\epsilon_r} = \frac{Q}{\epsilon_0 A \epsilon_r} \quad \text{--- (1)}$$

The field due to the charges $Q_p, -Q_p$ is directed oppositely & has magnitude,

$$E_p = \frac{\sigma_p}{\epsilon_0} = \frac{Q_p}{A\epsilon_0}$$

$$\text{The Resultant Field is } E = E_0 - E_p = \frac{Q - Q_p}{A\epsilon_0} \quad \text{--- (2)}$$

$$\text{From (1) \& (2): } \frac{Q - Q_p}{A\epsilon_0} = \frac{Q}{\epsilon_0 A \epsilon_r} \Rightarrow Q - Q_p = \frac{Q}{\epsilon_r} \Rightarrow \boxed{Q_p = Q\left(1 - \frac{1}{\epsilon_r}\right)}$$

ACTION OF SHARP POINTS:-

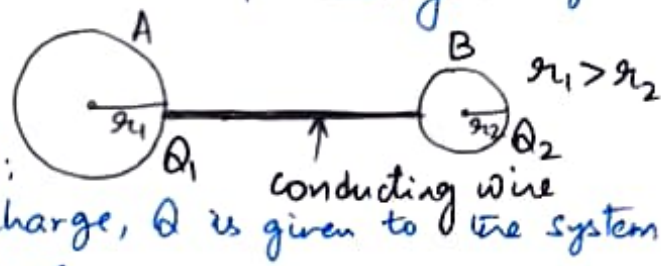
$$\text{For a spherical conductor: } \sigma = \frac{Q}{4\pi R^2}$$

for a pointed end: R -small $\Rightarrow \sigma \rightarrow$ very large.

The particles of air coming in contact with pointed ends get similarly charged & are repelled. In this way, an electric wind is set up which takes away the charge continuously from the pointed ends

of the conductor. This process of spraying the charge is called CORONA DISCHARGE. That is why, conductors used for storing charge are always spheres of large radii.

Consider two conducting spheres A & B of radii r_1 & r_2 resp; connected by a conducting wire. A charge, Q is given to the system. Let Q_1 resides on the surface of A & Q_2 on B.



$\therefore$ The pot. of sphere A, $V_1 = \frac{Q_1}{4\pi\epsilon_0 r_1}$ & that of sphere B, $V_2 = \frac{Q_2}{4\pi\epsilon_0 r_2}$

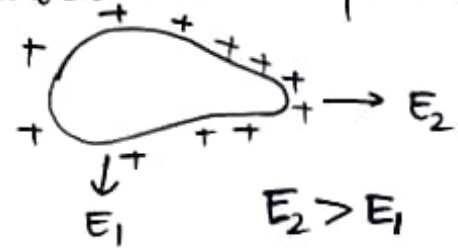
But $V_1 = V_2$ ($\because$ 2 spheres are connected by conducting wire)

$$\Rightarrow \frac{Q_1}{4\pi\epsilon_0 r_1} = \frac{Q_2}{4\pi\epsilon_0 r_2}$$

or $\sigma_1 r_1 = \sigma_2 r_2$ ($\because \sigma = \frac{Q}{4\pi r^2}$)

$$\Rightarrow \boxed{\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}} \Rightarrow \text{Sphere with smaller radius has larger charge density to maintain the same potential}$$

Now, consider a single conductor with non-spherical shape. The charge density will be larger, where the surface is more sharply curved or radius of curvature is small.



The Electric Field near the pointed ends will be very high which may cause dielectric breakdown in air. The charge may jump from the conductor to the air because of increased conductivity of the air. Often this discharge of air is accompanied by a visible glow surrounding the pointed end. This phenomenon is called CORONA DISCHARGE.