SHORT NOTES: CLASS 12

CHAPTER 10: WAVE OPTICS

CORPUSCULAR THEORY (Sir Isaac Newton): It states that "light consists of extremely light and tiny particles known as corpuscles which travel with very high speeds from the source of light".
Using this theory, Newton was able to explain reflection and refraction but **failed to explain** the cause of interference, diffraction and polarization. The **major failure** of Newton's corpuscular theory was that it could not explain why the velocity of light was lesser in the denser medium compared to the vacuum.
HUYGENS WAVE THEORY: Christopher Huygens proposed his wave theory of light in the early 18th century. According to Huygens's theory, **light consists of waves** that travel through a very dilute and highly elastic material medium present everywhere in space". This medium is called *ether*.
As the medium is supposed to be very dilute and highly elastic, its density would be very low, and the modulus of elasticity would be very high so that the speed of the light would be very large.
Waves generally do not move in a straight line and they have got certain characteristics that allow them to move in all directions. http://www.physicsinduction.com

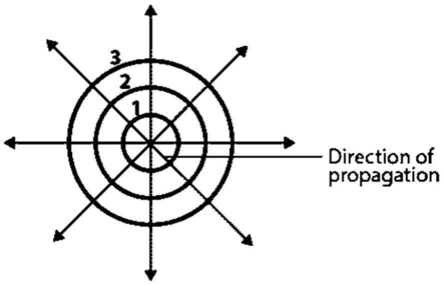
Outcomes of Huygens Principle

- The Huygens wave principle proved the concept of reflection of light, the concept of refraction of light, the concept of interference of light, and the concept of diffraction of light.
- It **failed to prove** the concept of polarization of light, emission of light, absorption of light, and the photoelectric effect.

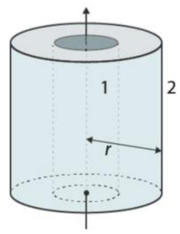
1. Polarization, as Huygens assumed that light waves, which are longitudinal in nature, are mechanical disturbances.
2. Black body radiation, photoelectric effect, and Compton effect.
3. Hypothetical medium *ether* was never discovered, and now we know light can propagate in a vacuum.

WAVEFRONT: A **wavefront** is defined as the locus of all points of the medium that vibrate in the same phase. Depending on the shape of the source of light, wavefronts can be of three types.

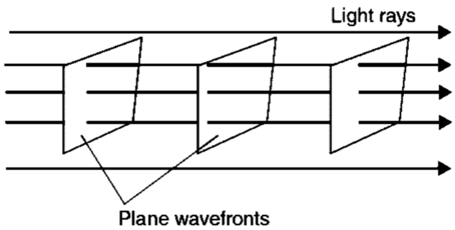
Spherical Wave Front: When light emerges from a point source, the wavefronts are spherical. http://www.physicsinduction.com
In spherical wavefront,
The amplitude of light waves, $A \propto 1/r$
The intensity of light waves, $I \propto 1/r^2$



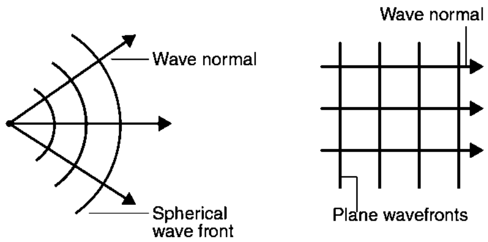
Cylindrical Wave Front: When the source of light is linear, the wavefronts are cylindrical. All the points are equidistant from the source.
The amplitude of light waves, $A \propto 1/\sqrt{r}$
The intensity of light waves, $I \propto 1/r$



Plane Wave Front
When the light is coming from a very far-off source, the wavefronts are planar. For a plane wavefront, the amplitude remains constant; therefore, the intensity remains constant.



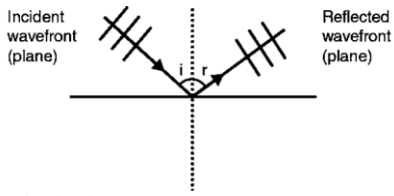
WAVE NORMAL: A perpendicular drawn to the surface of a wavefront at any point in the direction of propagation of light is called "wave normal". The direction in which light travels is called a 'ray' of light. Thus, a wave normal is the same as a ray of light.



Wavefronts for Reflection:

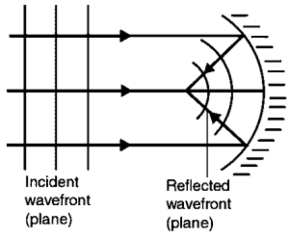
If light falls on a plane mirror

If the plane wavefronts are being reflected on the plane mirror, the shape of the wavefront of the reflected light is again planar.



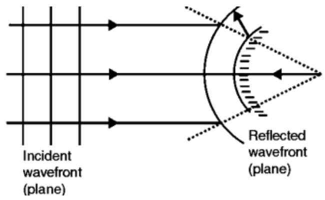
If light falls on a concave mirror

If a plane wavefront falls on a concave mirror, the shape of the reflected light is spherical.



If light falls on a convex mirror

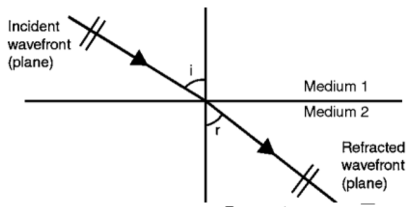
If a plane wavefront falls on a convex mirror, the shape of the reflected light is spherical.



Wavefronts for Refraction:

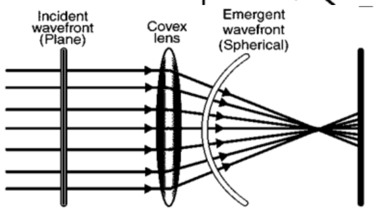
If light falls on plane surfaces

If a plane wavefront falls on a plane surface, the refracted ray will also have a plane wavefront.



If light falls on curved surfaces

If a plane wavefront falls on a converging (or) diverging lens, the emergent light will have a spherical wavefront.



http://www.physicsinduction.com

In 1678, Huygens suggested that each point on a wave's wavefront was a source of a spherical wave. The resultant wave is determined by adding all waves from the point of sources.

Huygens Principle of Secondary Wavelets: http://www.physicsinduction.com

The Huygens principle, also sometimes called the Huygens-Fresnel principle, states that each point on a given wavefront is a source of secondary wavelets or secondary disturbances. Further, the disturbances originating from the secondary source spreads in all directions in the same way as originating from the primary source.

This principle further highlights things such as;

- Secondary sources start making their own wavelets, these waves are similar to of primary source.
- At any instant of time, a common tangent on the wavelets in the forward direction gives the new wavefront.
- The sum of spherical wavelets forms the wavefront.

Example of Huygen's Principle

- If a stone is thrown into the river it will create waves around that point.
- These waves look like circular rings and are called wavefront waves.
- Gradually, the wavefronts disperse in all directions.
- When the locus of all the waves in the same phase is joined, it is the same as a sphere and is known as t he primary wavefront.
- Secondary wavefront is generated from each point on the primary wavefront.

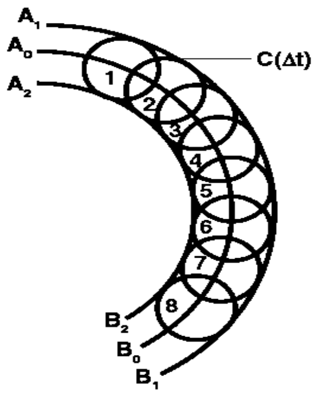
Moreover, the rising tangential line enveloping these secondary wavefronts can give rise to other secondary wavefronts. http://www.physicsinduction.com

- After some time all the wavefronts will slowly disappear.

Huygen's Theory Important Considerations

- All the rays are always perpendicular to the wavefront.
- Time taken by a wave from one wavefront to another wavefront is always constant. In the case of different mediums, distance and velocity can change but time will always remain constant.
- All the points on a wavefront act like a secondary source known as secondary wavelets.

Huygens Principle



- According to Huygens's principle, every point on a given wavefront can be regarded as a fresh source of new disturbance and sends out its own spherical wavelets called secondary wavelets. These secondary wavelets spread out in all directions with the velocity of the wave.
- A surface touching these secondary wavelets tangentially in the forward direction at any instant (Δt) gives the position and shape of the new wavefront at the instant. This is called "secondary wavefront".

Huygens's principle of secondary wavelets **could not explain** why wavefronts of secondary wavelets are formed in the forward direction and not in the backward direction. http://www.physicsinduction.com

REFLECTION OF PLANE WAVES USING HUYGENS' PRINCIPLE: The law of reflection (i = r) can be derived using the wave theory.

AB: plane wavefront incident on the mirror.

Every point on AB is a source of secondary wavelets.

A'B': secondary wavefront formed after t seconds.

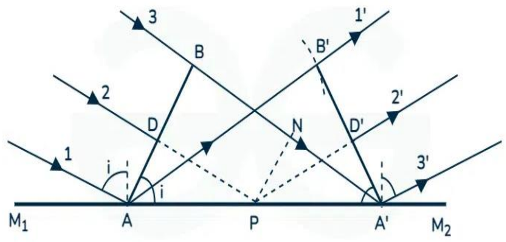
As The time taken by a wave from one wavefront to another wavefront is always constant.

Therefore, the time taken from A to B' is the same as the time taken from A to B'.

Velocity remains the same in a given medium.

⇨ AB' = c x t and BA' = c x t

Consider ΔABA' and ΔAB'A':



AB' = BA' = c x t

∠ABA' = ∠A'B'A (each 90°)

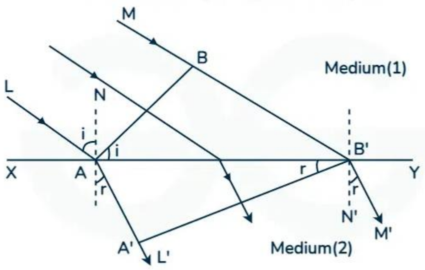
AA' = A'A (common)

ΔABA' is congruent to ΔAB'A' (RHS Congruence Rule)

Therefore: ∠ i = ∠r (CPCT)

REFRACTION OF PLANE WAVES USING HUYGENS' PRINCIPLE: Snell's law of refraction can be derived using the wave theory.

- Consider a plane wavefront AB incident on a surface XY separating two media 1 and 2.
- The medium 1 is a rarer medium of refractive index n_1 in which light travels with a velocity c_1.
- The medium 2 is the denser medium of refractive index n_2 in which light travels with a velocity c_2.
- The angle between the incident ray LA and the normal NA at the point of incidence A is equal to i.



- The angle is also equal to the angle between the incident plane wavefront AB and the surface of separation XY. So ∠BAB' is the angle of incidence of the incident plane wave front AB.
- Similarly, the angle between the refracted wavefront A'B' and the surface of separation XY is equal to the angle of refraction r. i.e. ∠AB'A'=r.

Consider ΔABB' and ΔAA'B' figure above.

sin i = BB'/AB' = c_1t/AB'   sinr = AA'/ AB' = c_2t/AB'

sin i/sin r = c_1t/ c_2t = c_1/ c_2

This constant is called the refractive index of the second medium (2) with respect to the first medium (1).

c_1/ c_2 = n_2/n_1 = n_{12}

COHERENT AND INCOHERENT SOURCES

Coherent Source: Two sources that emit a monochromatic light continuously with a zero (or) constant phase difference between them are called coherent sources. http://www.physicsinduction.com

Incoherent Source: The sources which do not emit light with constant phase difference are called incoherent sources.

RELATION BETWEEN FREQUENCY AND SPEED: The frequency remains the same as light travels from one medium to another. The speed v of a wave is given by $v = \frac{\lambda}{T}$

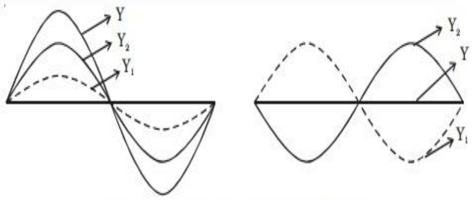
Where λ is the wavelength of the wave and $T(= 1/v)$ is the period of oscillation

DOPPLER EFFECT: Whenever there is a relative motion between the source and observer then the apparent frequency of light received by the observer is different from the actual frequency emitted by the source of light. This effect is called Doppler's effect in light. The effect can be used to measure the speed of an approaching or receding object.

Change in Frequency: For the source moving away from the observer $v < v_0$, and for the source moving towards the observer $v > v_0$, The change in frequency is $\Delta v = v - v_0 = -\frac{v}{c}v_0$

So, finally, $\frac{\Delta v}{v_0} = -\frac{v}{c}$

SUPERPOSITION PRINCIPLE: When two or more wave motions travelling through a medium superimpose one another, a new wave is formed in which resultant displacement, **y** at any instant is equal to the sum of the displacements (**y_1, y_2, ………. y_n**) due to individual waves at that instant.



$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} + \ldots\ldots\ldots + \vec{y_n}$

INTERFERENCE OF LIGHT: The effect of non-uniform energy distribution in the medium due to two superimposed light waves is called interference.

It is the phenomenon of redistribution of light energy in a medium on account of the superposition of light waves from two coherent sources.

CONDITIONS FOR SUSTAINED INTERFERENCE: http://www.physicsinduction.com

i. The two light sources must be **coherent**: i.e., they should emit continuous light waves of the same wavelength, same frequency, and in the same phase.

ii. The light sources should be **monochromatic**: If the sources produce white light consisting of a number of wavelengths then the light of each wavelength gives its own set of interference fringes.

Therefore, overlapping of fringes occurs. These fringes are hazy whereas those with monochromatic light are quite sharp.

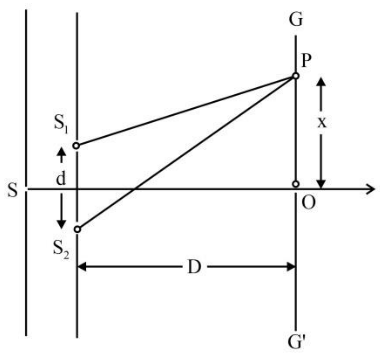
iii. The two light sources should be **point sources**.

iv. The **amplitudes** should be either equal or very nearly **equal**.

v. The **slit sources** should be **narrow and close** to each other.

YOUNG'S DOUBLE SLIT EXPERIMENT http://www.physicsinduction.com

Two parallel and very close slits S_1 and S_2 (illuminated by another narrow slit) behave like two coherent sources and produce a pattern of dark and bright bands on a screen known as interference fringes.

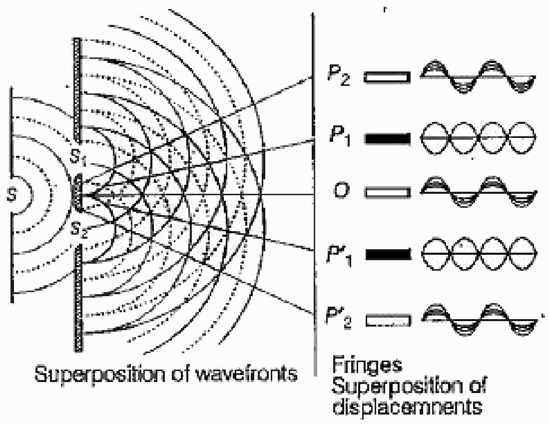


GG' is the screen, alternate bright and dark fringes are appeared on this screen.

As, according to Huyghen's principle, a monochromatic source of light sends out spherical wavefronts. Let solid arcs represent crests and dotted arcs represent troughs. On reaching slits S_1 and S_2, these wavefronts become the sources of secondary wavelets. Thus, the two waves of the same frequency and same amplitudes are produced.

Here dots represent the position of constructive interference and crosses represent the position of destructive interference.

In case of constructive interference, the crest of one wave falls on the crest of the other and the trough falls on the trough.



Similarly, In the case of destructive interference, the crest of one wave falls on the trough of the other and vice versa. The resultant amplitude and hence the intensity of light is minimal at these positions. Thus, bright and dark fringes are produced. They are placed alternately and are equally spaced. These bands are also called **interference fringes**.

http://www.physicsinduction.com

CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE:

Let the waves from two coherent sources be represented as:

$Y_1 = a \sin\omega t$ and $Y_2 = b \sin(\omega t + \phi)$

Where a and b are the respective amplitudes and ϕ is the phase angle.

According to the superposition principle,

Displacement of the resultant wave at time t

$Y = Y_1 + Y_2$

⇨ $Y = a \sin\omega t + b \sin(\omega t + \phi)$

⇨ $Y = a \sin \omega t + b \sin\omega t \cos\phi + b \cos\omega t \sin\phi$

⇨ $Y = \sin \omega t(a+ b \cos\phi) + \cos\omega t.\ b \sin\phi$

Let $(a+ b \cos\phi) = R\cos\theta$ and……………………………………………………………(i)

$b \sin\phi = R\sin\theta$……………………………………………………………………………(ii)

⇨ $Y = R(\cos\theta \sin \omega t + \sin\theta \cos\omega t)$

⇨ $Y = R \sin(\omega t + \theta)$

∵ Resultant wave is a harmonic wave of amplitude R and phase angle θ.

Squaring and adding (i) and (ii), we get

$R^2(\cos^2\theta + \sin^2\theta) = (a+ b \cos\phi)^2 + b^2\sin^2\phi$

⇨ $R^2 = a^2 + b^2 (\sin^2\phi + \cos^2 \phi) + 2ab \cos\phi$

⇨ $R^2 = a^2 + b^2 + 2ab\cos\phi$

⇨ $R = \sqrt{(a^2 + b^2 + 2ab\cos\phi)}$

As, intensity α $(\text{resultant})^2$

⇨ $I \propto R^2$

⇨ $I \propto (a^2 + b^2 + 2ab\cos\phi)$

For constructive interference: intensity should be the maximum

http://www.physicsinduction.com

I is maximum, iff, $\cos\phi = 1$

⇨ $\phi = 0, 2\pi, 4\pi$……….

⇨ $\phi = 2n\pi$

let X be the path difference between two waves, then

as, $X = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi} \times 2n\pi$

⇨ $X = n\lambda$

$R_{max} = a + b$ when $\cos\phi = 1$

& $I_{max} = k(a + b)^2$

For destructive interference: intensity should be the minimum

I is minimum, iff, $\cos\phi = -1$

⇨ $\phi = \pi, 3\pi, 5\pi$……….

⇨ $\phi = (2n-1)\pi$

let X be the path difference between two waves, then http://www.physicsinduction.com

as, $X = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi} \times (2n-1)\pi$

⇨ $X = (2n-1)\lambda/2$

$R_{min} = a - b$ when $\cos\phi = -1$

& $I_{min} = k(a - b)^2$

$I_{max}/I_{min} = (a + b)^2/(a - b)^2$

When $b = a$: $R_{max} = 2a$

And $R_{min} = 0$

⇨ $I_{min} = 0$: Perfectly dark

Also, Resultant Intensity of individual intensities I_1 and I_2 due to the two sources may be written as: (take $a^2 = I_1$, $b^2 = I_2$ and $R^2 = I$ in $R^2 = a^2 + b^2 + 2ab\cos\phi$)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

The interference term averaged over many cycles is zero if

1. The sources have different frequencies or
2. The sources have the same frequency but no stable phase difference.
3. For such coherent sources-$I = I_1 + I_2$
4. The resultant intensity depends on the relative location of the point from the two sources, since changing it changes the path difference as we go from one point to another.
5. As a result, the resulting intensity will vary between maximum and minimum values, determined by the maximum and minimum values of the cosine function.

These will be = $\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Width is directly proportional to the intensity

Let w_1 and w_2 be the widths of the two slits http://www.physicsinduction.com

Therefore, $w_1/w_2 = I_1/I_2 = a^2/b^2$

INTERFERENCE AND ENERGY CONSERVATION

$I_{max} = k(a + b)^2$ and $I_{min} = k(a - b)^2$

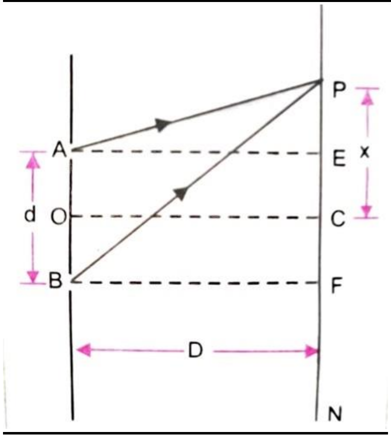
$I_{average} = (I_{max} + I_{min})/2 = k(a^2 + b^2)$

If there were no interference, the intensity of light from two sources at every point on the screen would be $I = I_1 + I_2 = k(a^2 + b^2)$, which is the same as $I_{average}$ in the interference pattern.

The intensity of light is simply being distributed. http://www.physicsinduction.com

⇨ Energy is only transferred from destructive to constructive. No energy is created or destroyed.

EXPRESSION FOR FRINGE WIDTH IN INTERFERENCE OR THEORY OF INTERFERENCE OF LIGHT:



d: separation between two slits,

D: distance between the slits and the screen and x is the distance of the point of *P* from the central fringe.

For a point P on the screen,

The path difference BP – AP……………………….(i)

AB = EF = d, AE = BF = D

PE = PC – EC = x – d/2

PF = PC + CF = x + d/2

In ΔBPF, $BP = [BF^2 + PF^2]^{1/2}$

$$BP = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right]^{1/2}$$

$$BP = D\left[1 + \left(\frac{x+d/2}{D}\right)^2\right]^{1/2}$$ http://www.physicsinduction.com

$BP = D\left[1 + \frac{(x+d/2)^2}{2D^2}\right]$ (by expanding binomially)…………………….(ii)

In Δ APE, $AP = [AE^2 + PE^2]^{1/2}$

$$AP = \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]^{1/2}$$

$$AP = D\left[1 + \left(\frac{x - d/2}{D}\right)^2\right]^{1/2}$$

$AP = D\left[1 + \frac{(x- d/2)^2}{2D^2}\right]$ (by expanding binomially)…………………….(iii)

From (i), (ii) and (iii)

Path difference = BP – AP = $D\left[1 + \frac{(x+d/2)^2}{2D^2}\right]$ - $D\left[1 + \frac{(x- d/2)^2}{2D^2}\right]$ = $\frac{D}{2D^2}\left[(x + d\,/\,2)^2 - \left(x - {}^{d}/_{2}\right)^2\right]$

= $\frac{1}{2D}\left(4x\frac{d}{2}\right)$ = $\frac{xd}{D}$

For bright fringes, (maxima). Path difference = nλ http://www.physicsinduction.com

Therefore, $\frac{xd}{D}$ = nλ

⇨ $x = \frac{n\lambda D}{d}$

For dark fringes, (minima). Path difference = $(2n - 1)\frac{\lambda}{2}$

Therefore, $\frac{xd}{D}$ = $(2n - 1)\frac{\lambda}{2}$

⇨ $x = (2n - 1)\frac{\lambda}{2}\frac{D}{d}$

FRINGE WIDTH:

Width of bright fringe: the separation between the centres of two consecutive dark fringes

$\beta = x_n - x_{n-1}$

⇨ β = $(2n - 1)\frac{\lambda}{2}\frac{D}{d}$ - $[2(n - 1) - 1]\frac{\lambda}{2}\frac{D}{d}$

⇨ β = $\frac{\lambda D}{d}$

Width of dark fringe: the separation between the centres of two consecutive bright fringes

$\beta' = x'_n - x'_{n-1}$

⇨ $\beta' = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$

⇨ $\beta = \frac{\lambda D}{d}$

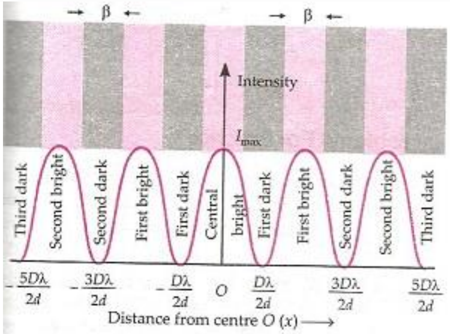
Hence, all bright dark fringes are of equal width. http://www.physicsinduction.com

For constructive interference (bright band), the path difference must be an integer multiple of λ , i.e.-

The separation β between adjacent bright (or dark) fringes is, $\beta = \frac{D\lambda}{d}$ using which λ can be measured.

Young's Double Slit Interference Experiment: Fringe width, $\beta = \frac{D\lambda}{d}$

Intensity distribution curve for interference:



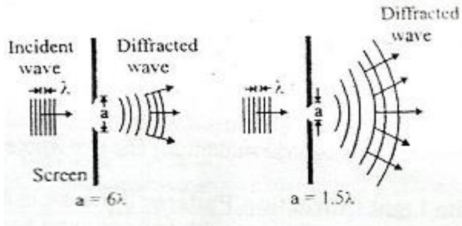
Interference fringes with white light:- When the slits are illuminated with white light, the interference pattern consist of a central white fringe having on both sides a few coloured fringes and then a general illumination.

Diffraction: The phenomenon of bending of light around the corners of an obstacle or aperture in the path of light is called the diffraction of light.

Diffraction can be observed readily in radio waves and sound waves because λ is large.

It is not so common in visble light because λ is small (≈ 10^{-6}m) therefore, diffraction is not so common.

Diffraction is most pronounced when the slit width is the smallest, i.e., a = 1.5λ.



Small aperture => better diffraction

According to Fresnel, diffraction occurs on account of mutual interference of secondary wavelets from the positions which are not allowed to pass through the aperture.

Fraunhofer's Diffraction due to Single Slit:

S: source of monochromatic light

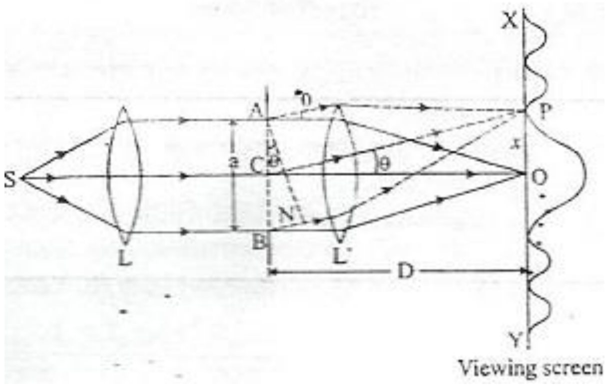
L: collimating lens

L': convex lens

AB: single slit of width a(order of λ of light)

XY: screen

WW': plane wavefront



Secondary waves from the points equidistant from centre C of the slit lying in the portions of CA and CB of wavefront travel the same distance in reaching O and hence, the path difference between them is zero.

Now, consider secondary waves travelling at an angle, θ with CO. Let all these secondary waves reach at point P on the screen. Draw AN ⊥ BK. http://www.physicsinduction.com

- Path difference between the secondary waves reaching P = BN = AB Sinθ = a Sinθ
- If this path difference is λ, then P will be the point of minimum intensity.
- For every point in the upper half AC, there is a corresponding point in the lower half CB for which the path difference between the secondary waves reaching P is λ/2.

If path difference = λ

⇨ P: Point of minimum intensity

For nth secondary minimum: Path difference = a $\sin\theta_n$ = nλ

First secondary maximum: $\lambda/2$ then $3\lambda/2$, $5\lambda/2$, $7\lambda/2$.............

In general, for nth secondary maximum: Path difference = a $\sin\theta_n = (2n + 1)\lambda/2$

Here n = 1, 2, 3,.........., an integer

Width of central maximum: The width of the central maximum is the distance between the first secondary minimum on either side of O. if P is the position of the first secondary minimum. Then,

a $\sin\theta = \lambda$(for first secondary minimum, n =1) http://www.physicsinduction.com

⇨ $\sin\theta = \lambda/a$

⇨ $\theta = \lambda/a$, if θ is very small.

If f is the focal length of the lens L' held very close to the slit. f ≈ D

Then $\theta = x/f = x/D$

⇨ $x/D = \lambda/a$

⇨ $x = \lambda D/a$

width of central maximum = 2x = $2\lambda D/a = 2\lambda f/a$

- The single-slit diffraction pattern shows the central maximum $(at\ \theta = 0)$, zero intensity at angular separation $\theta = \pm(n + 1/2)\lambda......(n \neq 0)$
- Angular spread of the central maxima = $\lambda D/a$

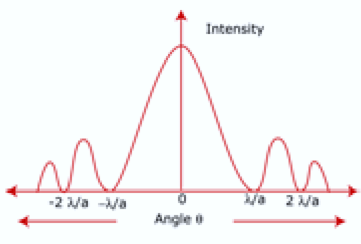
  Width of the central maxima: $2\lambda D/a$ http://www.physicsinduction.com

  Where D is the distance of the slit from the screen, a is the slit width.
- Width of the central maximum is directly proportional to the wavelength of light and inversely proportional to the width of the slit.
- S: monochromatic light: central maximum- maximum intensity, all bright and dark bands of unequal widths and decreasing intensity.

  S: white light : pattern–coloured. Central maximum – white. Bandwidth α λ, therefore, red band(higherλ)- wider than violet band (smaller λ)

The intensity plot looks as follows, with there being a bright central maximum, followed by smaller intensity secondary maxima, with there being points of zero intensity in between, whenever

a $\sin\theta_n = n\lambda$, $n \neq 0$



Relation between phase difference & path difference:

Phase difference = $(2\pi/\lambda)$ Path difference

$$\Delta\phi = \frac{2\pi}{\lambda}.\Delta X$$

Where $\Delta\phi$ is the phase difference & ΔX is the path difference http://www.physicsinduction.com

$\Delta\phi = (2\pi/\lambda)\, a \sin\theta$

Interference Vs Diffraction:

	INTERFERENCE	DIFFRACTION
1.	Superposition of two waves: 2 sources	Superposition of secondary wavelets
2.	All bright fringes: same intensity	Different intensity: intensity decreases
3.	Intensity of minima ≈ 0	The intensity of minima is never 0
4.	Good contrast between bright and dark fringes	Poor contrast
5.	The width of the fringes may/may not be equal.	Always unequal