PHYSICS INDUCTION

An institute of Science & Mathematics

Vinita Chawla
M.Sc. Physics (HRC, Delhi University)
M.Sc. Computer Science, B.Ed.,
CTET, HTET, PSTET, CSB Qualified
Contact: 91-8168258349
physicsinduction@gmail.com

CLASS XII : NOTES : CHAPTER-1 : ELECTRIC CHARGES AND FIELDS : PHYSICS

ELECTROSTATICS/STATIC ELECTRICITY:- The Branch of Physics, which deals with the study of electric charges at rest.

ELECTRIC CHARGE:- Electric charge is a fundamental physical quantity due to which electrical and other related effects are produced in matter. It is a form of energy and it can neither be created nor be destroyed.

Unit of Electric Charge:- Charge of an object is a measurable quantity. The charge possessed by an object is measured in 'Coulomb', C. C.G.S. unit of charge is statcoulomb.

$$\boxed{1C = 3 \times 10^9 \text{ statcoulomb}}$$

Types of Electric charge:-

(i) Positive charge:- Positively charged objects possess more protons than electrons. Loss of electrons occurs, therefore mass reduces.

(ii) Negative charge:- Negatively charged objects possess more electrons than protons. Gain of electrons occur, therefore, mass increases.

$q(\text{Proton}) = +1.6 \times 10^{-19}\,C$, $\quad q(\text{Electron}) = -1.6 \times 10^{-19}\,C$

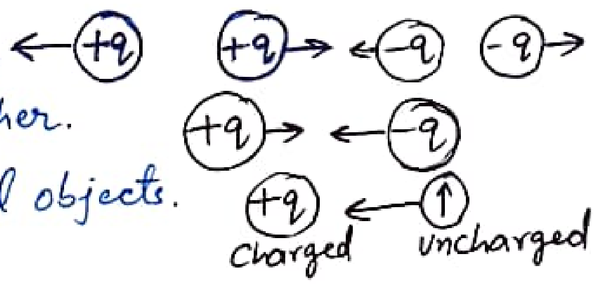
* The quantity of charge depends upon the number of excess e^- or p present in the object.

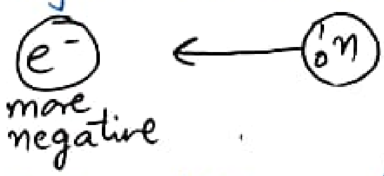
Charge Interactions :-

(i) Like charges repel each other.

(ii) Unlike charges attract each other.

(iii) charged objects attract neutral objects.





(iv) If $Q >> q$, then

Apparatus used to detect charge on a body :- Gold-Leaf Electroscope

Principle :- It works on the principle that like charges repel each other.

Working :- It consists of a metal rod. At one end of the rod, two gold leaves are fixed. At the other end, there is a metal disc. When the metal disc is touched with a charged body. The gold leaves will move away from each other. Because, charges are transferred to the leaves by conduction. And, nature of charges on both the leaves is similar. Hence, they would repel each other.

Properties of Electric charge :-

(i) **Quantization of Charge** :- Electric charge on a body is an integral multiple of charge on an electron.

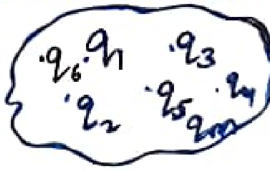
i.e., $q = \pm ne$, where $n = 1, 2, 3, ---$ — 1st suggested by Faraday, further established by Millikan.

(ii) **Conservation of charge** :- For an isolated system, charge is always conserved. It can neither be created, nor be destroyed. It can only be transferred from one body to another.

(iii) **Invariance of charge** :- Charge invariance refers to the fixed value of charge regardless of its speed. It's independent of speed.

(iv) **Additivity of charge** :- For an isolated system, the net charge is the algebraic sum of all the charges present in the system.

$$\text{Total charge, } q = q_1 + q_2 + ---- + q_n$$
$$= \sum_{i=1}^{n} q_i$$

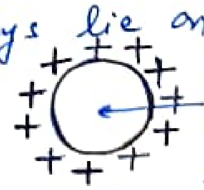


If a system contains n, no. of charges

* charge is a scalar quantity.

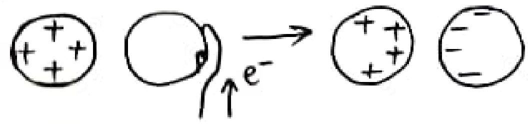
Note :-

- No q resides inside a conductor, ±q always lie on the outer surface of a conductor



Can be hollow or solid

- **Ground** :- infinite source of Electron.
  source → supply e^-
  OR
  sink for Electron
  sink ← receive e^-



Ways of Charging :

Charging
- by Friction
- by Induction
- by conduction

(i) Charging by Friction / Frictional Electricity :-

→ when two objects are rubbed against each other, then the one with greater affinity for electron pull electrons from the object with lower electron affinity.

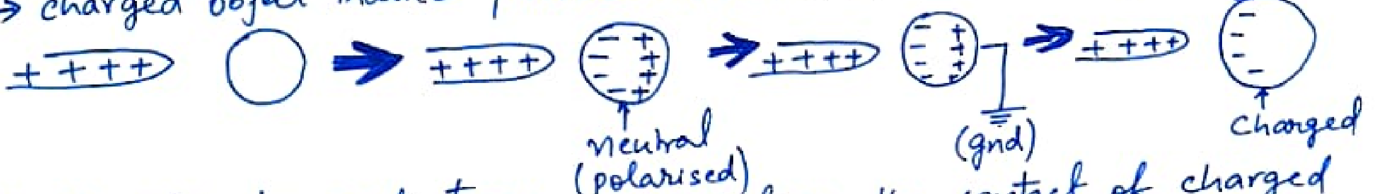
→ This method follows the law of conservation of charges.

- Before Rubbing : both the objects are neutral.
- After Rubbing : One acquires +ve q, other acquires equal −q.

(ii) Charging by Induction :-

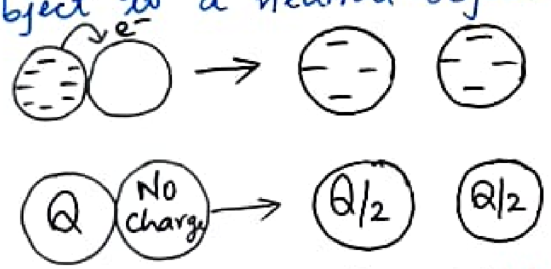
→ It's a method used to charge an object without actually touching it.

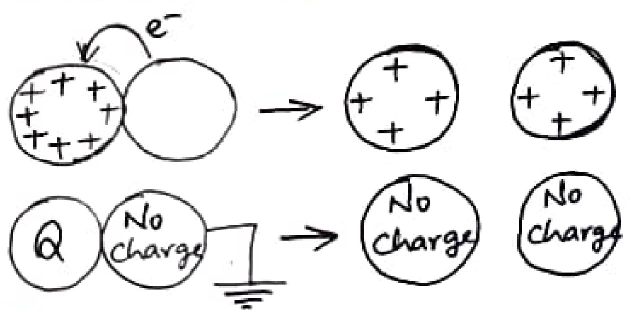
→ charged object induce polarization within the neutral object.



(iii) Charging by conduction :-

It involves the contact of charged object to a neutral object.





CONDUCTORS AND INSULATORS :-

Conductors :- Conductors are those substances, which allow the electricity to pass through them easily. They have free electrons in them, which are responsible for the conduction of current. e.g. Metals, Human, Animal Body, Earth etc.

Insulators :- Insulators are those substances, which offer resistance to the passage of electricity. They do not have free e^- in them. e.g., glass, porcelain, plastic, nylon, wood.

COULOMB'S LAW :-

* **point charges :** linear size of charged bodies << distance b/w the charged bodies.

Acc. to this law, the force of attraction (repulsion) b/w two pt. charges is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance b/w them.

consider two charges, q_1 and q_2, separated by a distance, r; then, according to Coulomb's Law,

The Force, F (attractive / repulsive) b/w the charges,

$$F \propto q_1 q_2 \qquad F \propto \frac{1}{r^2}$$

Like qs : Repulsion : F = +ve
Unlike qs : Attraction : F = −ve

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = \frac{K q_1 q_2}{r^2} \quad \text{where, } K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/C^2.$$

where, ε_0 is the permittivity of free space/vacuum, degree to which a medium can resist the flow of q.

For $r < 10^{-15}$ m, Coulomb's law is not valid.

1 Coulomb :- 1 Coulomb is that much charge, which when placed in vacuum at a distance of 1m from an equal and similar charge would repel it with a force of 9×10^9 N.

$q_1 = q_2 = 1C$, $r = 1m$

$$\text{As, } F = \frac{K q_1 q_2}{r^2} \Rightarrow F = \frac{9\times10^9 \times 1 \times 1}{1^2} = 9\times10^9 N$$

e-p system

e —r→ p

$$\frac{F_E}{F_G} = \frac{Ke^2}{G m_e m_p} \simeq 2.4\times10^{39}$$

Relative Electrical Permittivity :-

As, $F = \frac{K q_1 q_2}{r^2}$ where, $K = \frac{1}{4\pi\varepsilon_0}$ where, ε_0 : permittivity of free space.

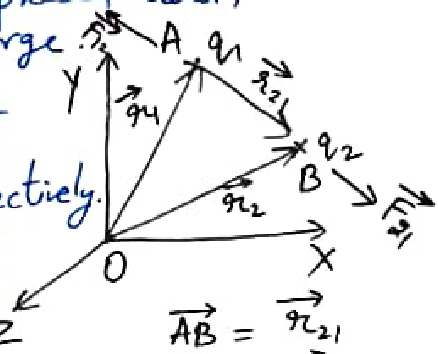
In a dielectric medium, $K = \frac{1}{4\pi\varepsilon}$ where, ε: permittivity of the medium.

Relative Electrical Permittivity : ε_r : $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ where, ε_r: Relative permittivity of the medium w.r.t. vac.

$$\therefore K = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$$

$\varepsilon_r = 1$ (vacuum), $\varepsilon_r = 1.00054 \simeq 1$ (air)
$\varepsilon_r = \infty$ (for conducting medium)

The magnitude of K depends upon:
(i) Units in which F, q_1, q_2 and r are expressed and,
(ii) Properties of the medium around the charge.



Coulomb's Law in Vector Notation :-

Consider two charges, q_1 and q_2 at pts A & B respectively.

$\vec{OA} + \vec{AB} = \vec{OB}$ (Triangle law of Vector Add^n)

$\Rightarrow \vec{r}_1 + \vec{r}_{21} = \vec{r}_2 \Rightarrow \vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

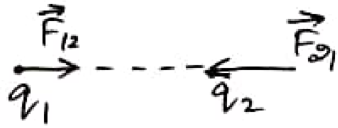
$\vec{AB} = \vec{r}_{21}$
$\vec{BA} = \vec{r}_{12}$

IIly, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$

$|\vec{r}_{12}| = |\vec{r}_{21}|$

$\hat{r}_{12} = -\hat{r}_{21}$

$q_1 q_2 > 0$

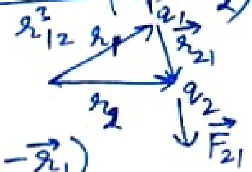


$q_1 q_2 < 0$

The Force on charge, q_1 due to charge, q_2 : $\vec{F}_{12}$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^3}\vec{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^3}(\vec{r}_1 - \vec{r}_2)$$

The Force on charge, q_2 due to charge, q_1 : $\vec{F}_{21}$

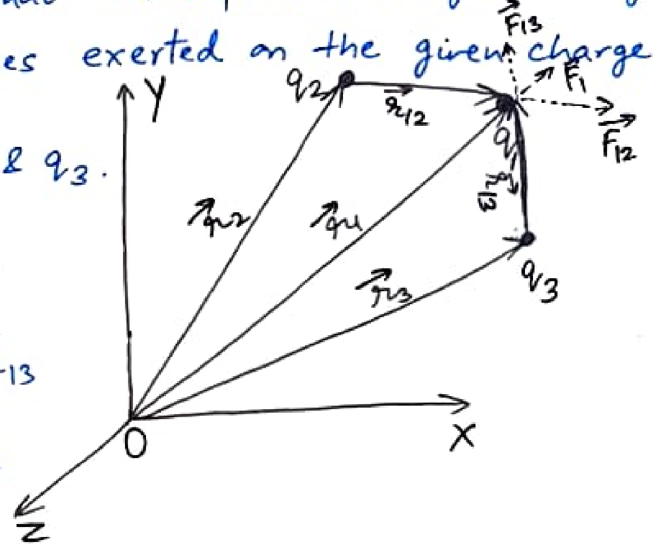


$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}(\vec{r}_2 - \vec{r}_1)$$

THE SUPERPOSITION PRINCIPLE :- Forces b/w Multiple charges

The principle of superposition states that "Total force on a given charge is the vector sum of individual forces exerted on the given charge by all the charges.

Consider, three charges q_1, q_2 & q_3.



Force on q_1 due to q_2 : $\vec{F}_{12} = K\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$

Force on q_1 due to q_3 : $\vec{F}_{13} = \frac{K q_1 q_3}{r_{13}^2}\hat{r}_{13}$

$\therefore$ Total Force, F_1 on q_1 : $\vec{F} = \vec{F}_{12} + \vec{F}_{13}$

$$\Rightarrow \vec{F}_1 = K q_1\left[\frac{q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_3}{r_{13}^2}\hat{r}_{13}\right]$$

for n charges : $F_1 = K q_1 \sum_{i=2}^{n} \frac{q_i}{r_{1i}^2}\hat{r}_{1i} = K q_1 \sum_{i=2}^{n}\frac{q_i}{r_{1i}^3}(\vec{r}_1 - \vec{r}_i)$

In general, total Force, $\vec{F}_0$ on a test charge q_0 at $\vec{r}_0$ due to all the n discrete charges :

$$\vec{F}_0 = K q_0 \sum_{i=1}^{n}\frac{q_i}{r_{0i}^2}\hat{r}_{0i} = K q_0 \sum_{i=1}^{n}\frac{q_i}{r_{0i}^3}(\vec{r}_0 - \vec{r}_i)$$

The Superposition Principle is based on the property that the forces, with which the two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s).

ELECTRIC FIELD :- The space around a charged body where Electric force is experienced by a test charge is called an Electric Field.

Electric Field strength :- The Electric Field strength or Electric Field Intensity implies the strength of the Electric Field around a charge.

The Electric field intensity at any point in an electric field is defined as the force per unit charge exerted on a tiny positive test charge at that point

$$\vec{E} = \lim_{q \to 0} \left(\frac{\vec{F}}{q} \right)$$ [∵ If the test charge is too big, it perturbs the field]

Unit :- N/C

$\vec{E}$ represents a Vector Quantity, whose direction is same as the force that would be experienced by a positive test charge.

$$\vec{F} = \frac{KQq}{r^2}\hat{r} \qquad \left(K = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\therefore \vec{E} = \frac{\vec{F}}{q} = \frac{KQ}{r^2}\hat{r}$$

$$\Rightarrow \boxed{\vec{E} = \frac{KQ}{r^3}\vec{r} \quad \| \quad \vec{F} = q\vec{E}}$$

field (3-D) (spherical symmetry)

Q : Source charge

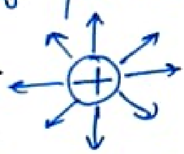
q : Test charge

small, doesn't affect the $\vec{E}$ of Q.

If q = Unity : $\vec{E}$ due to a charge, Q at a point in space may be defined as the force that a unit positive charge would experience, if placed at that pt. $\boxed{q = 1C \Rightarrow \vec{E} = \vec{F}}$

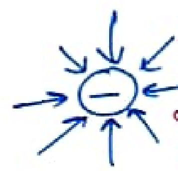
+ve charge

$\vec{E}$: Radially outwards



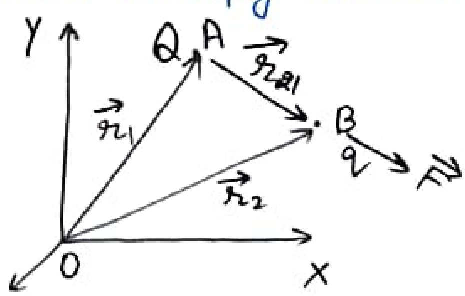
-ve charge :

$\vec{E}$: Radially inwards.



Note :-
- $\vec{E}$: independent of q
- $\vec{E}$: depends on Q and space coordinate, $\vec{r}$
- $\vec{E}$: exists at every pt in 3-D space
- $\vec{E}$: more where surface is sharply curved.

$\vec{E}$ due to a point charge, q :



$$\vec{F} = \frac{KQq}{r_{21}^2}\hat{r}_{21} \qquad \left[K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{KQ}{r_{21}^2}\hat{r}_{21} = \frac{KQ}{r_{21}^3}\vec{r}_{21}$$

$$\Rightarrow \boxed{\vec{E} = \frac{KQ}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)}$$

The field takes finite time to propagate. Thus, if a charge is displaced from its posⁿ, the field at a distance, r will change after time, t = r/c, where, c is the speed of light.

$\vec{E}$ due to a system of charges :- The Electric field intensity at any point, P due to a system of charges ($q_1, q_2, ..., q_n$) is equal to the sum of the electric fields exerted by the individual charges at point, P.

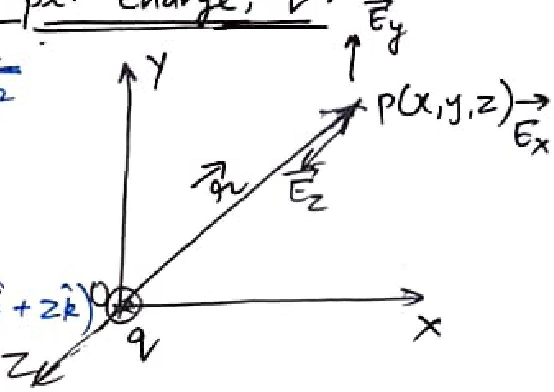
$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \text{-------} + \vec{E}_n(\vec{r})$$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2p}^2}\hat{r}_{2p} + \text{-----} + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{np}^2}\hat{r}_{np}$$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^2}\hat{r}_{ip} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^3}\vec{r}_{ip}$$

$$\boxed{\vec{E}(\vec{r}) = k\sum_{i=1}^{n}\frac{q_i}{|r_i - r_p|^3}(\vec{r}_i - \vec{r}_p)} \quad \left[\text{where, } k = \frac{1}{4\pi\varepsilon_0}\right]$$

Rectangular components of $\vec{E}$ due to a pt. charge, q :-



$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{r}| = \sqrt{x^2+y^2+z^2}$

$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

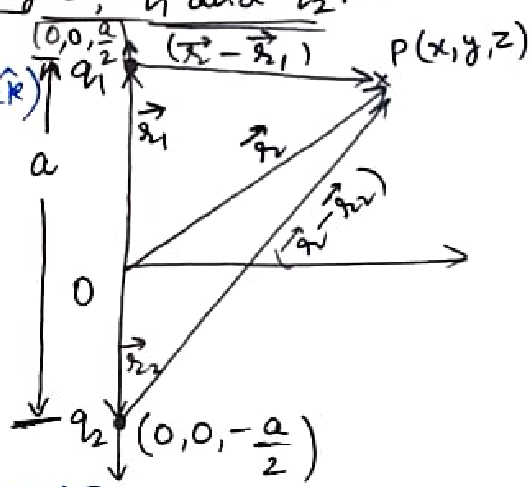
As, $\vec{E} = \frac{kq}{r^3}\vec{r}$

$$\Rightarrow E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = \frac{kq}{(x^2+y^2+z^2)^{3/2}}(x\hat{i} + y\hat{j} + z\hat{k})$$

By comparing coeff., we get.

$$E_x = \frac{kqx}{(x^2+y^2+z^2)^{3/2}} \; ; \; E_y = \frac{kqy}{(x^2+y^2+z^2)^{3/2}} \; ; \; E_z = \frac{kqz}{(x^2+y^2+z^2)^{3/2}}$$

$\vec{E}$ (at a pt) due to a system of two point charges, q_1 and q_2 :-



$\vec{r}_1 = \frac{a}{2}\hat{k}$, $\vec{r}_2 = -\frac{a}{2}\hat{k}$, $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$

$\vec{r} - \vec{r}_1 = x\hat{i} + y\hat{j} + (z - \frac{a}{2})\hat{k}$

$\vec{r} - \vec{r}_2 = x\hat{i} + y\hat{j} + (z + \frac{a}{2})\hat{k}$

$$\vec{E}(\text{at } P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1(\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + \frac{q_2(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3}\right]$$

$$\vec{E}(\vec{r}) = k\left[\frac{q_1[x\hat{i} + y\hat{j} + (z - \frac{a}{2})\hat{k}]}{[x^2+y^2+(z-\frac{a}{2})^2]^{3/2}} + \frac{q_2[x\hat{i} + y\hat{j} + (z + \frac{a}{2})\hat{k}]}{[x^2+y^2+(z+\frac{a}{2})^2]^{3/2}}\right]$$

www.physicsinduction.com

$$\therefore E_x = k\left[\frac{q_1 x}{[x^2+y^2+(z-\frac{a}{2})^2]^{3/2}} + \frac{q_2 x}{[x^2+y^2+(z+\frac{a}{2})^2]^{3/2}}\right]$$

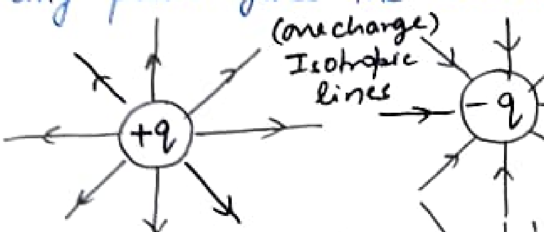
$$E_y = k\left[\frac{q_1 y}{[x^2+y^2+(z-\frac{a}{2})^2]^{3/2}} + \frac{q_2 y}{[x^2+y^2+(z+\frac{a}{2})^2]^{3/2}}\right]$$

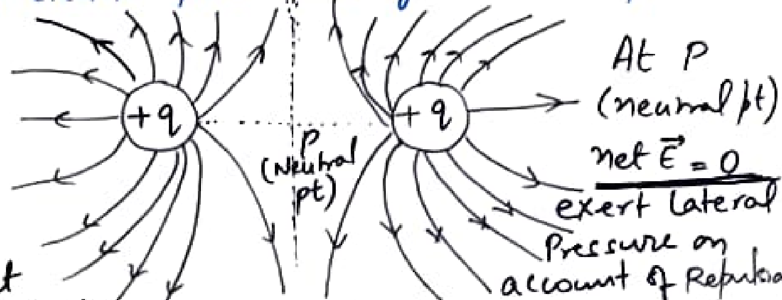
$$E_z = k\left[\frac{q_1(z-\frac{a}{2})}{[x^2+y^2+(z-\frac{a}{2})^2]^{3/2}} + \frac{q_2(z+a/2)}{[x^2+y^2+(z+\frac{a}{2})^2]^{3/2}}\right]$$

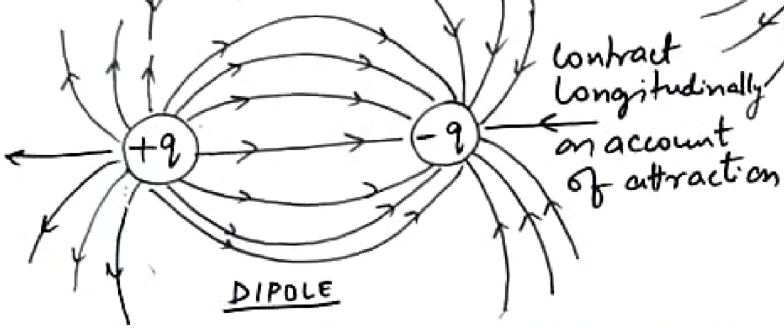
(Purely a geometrical construction, No physical existence)

ELECTRIC FIELD LINES : invented by Michael Faraday.

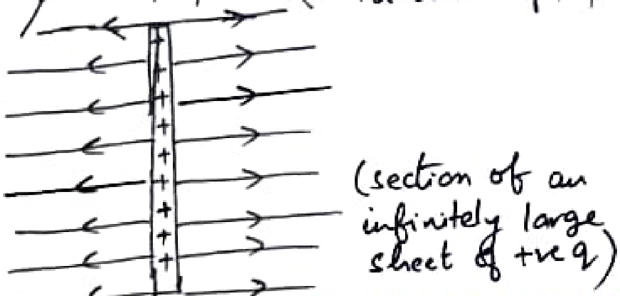
Electric field lines are a way of pictorially mapping the electric field around a configuration of charges. We may define, an electric line of force, as the path, straight or curved, such that tangent to it at any point gives the direction of Electric field intensity at that point.



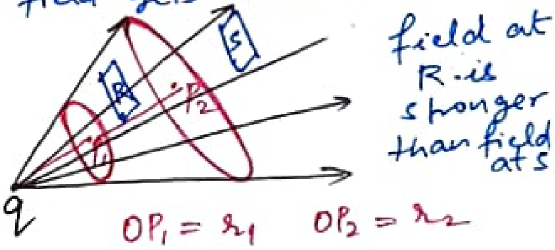




DIPOLE



- Magnitude of $\vec{E}$ is represented by the density of field lines. Near the charge, density of the field lines is more $\therefore$ $\vec{E}$ is strong. Away from the charge, field gets weaker, shown as well-separated field lines.



solid $\angle$, $\Delta\Omega = \dfrac{\Delta S}{r^2}$

$OP_1 = r_1$ $\quad OP_2 = r_2$

area subtended by solid $\angle$ at $P_1 = r_1^2 \Delta\Omega$

" " " " " " $P_2 = r_2^2 \Delta\Omega$

No. of lines (n) cutting these area elements are the same.

$\therefore$ No. of field lines, cutting unit area at $P_1 = \dfrac{n}{r_1^2 \Delta\Omega}$

" " " " " " " $P_2 = \dfrac{n}{r_2^2 \Delta\Omega}$

$\Rightarrow$ $\boxed{\text{strength of field has } 1/r^2 \text{ dependence.}}$

Note :- • $\vec{E}$ at a pt is equal to the number of field lines crossing normally a unit area around that pt. $\vec{E} \propto \dfrac{\text{No. of field lines}}{\text{Cross-sectional Area}}$

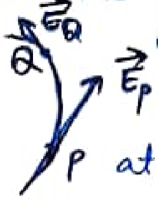
- $\rightrightarrows$ represents uniform $\vec{E}$
- Electric field lines are $\perp$r to the surface of the charged body
- $\perp$r to the equipotential surface.

Props of Field lines :-

(i) For single q, Field lines may start or end at infinity. For two opp qs, +q & -q, field lines start from +qs & end at -qs

(ii) In a charge free region, Electric field lines can be taken to be continuous curves, without any breaks.

(iii) Tangent to a field line at any pt, gives the direction of $\vec{E}$ at that pt. Two field lines can never cross each other. because at the pt of intersection, we can draw 2 tangents at the same pt, which This would mean, two direct^n of $\vec{E}$, is not possible.

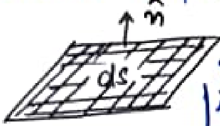


(iv) field lines don't form any closed loops.

ELECTRIC FLUX :-

Area Vector :- $d\vec{s} = ds\,\hat{n}$

ds - small area element



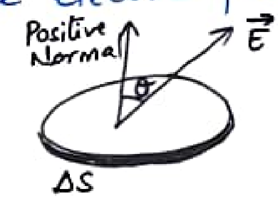
An area vector is a vector, whose magnitude is equal to the area of a plane & whose direct^n is normal to the plane of the area.

The Vector associated with every area element of a closed surface is taken to be in the direct^n of the outward normal

Electric Flux :- Electric Flux is the measure of the number of field lines passing through the surface.

Electric Flux over an area represents the electric field lines over that area.

$$\phi_E = \sum_{\text{all area element}} \vec{E}.d\vec{s} = \int_s \vec{E}.d\vec{s}$$



(i) It is a scalar quantity.

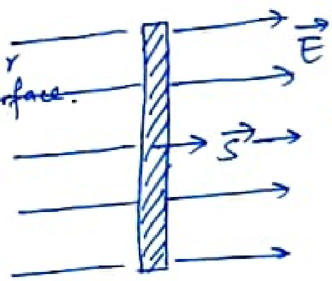
(ii) Unit : Nm^2/C.

Uniform Field :- $\phi = \vec{E}.\vec{s} = Es\cos\theta$

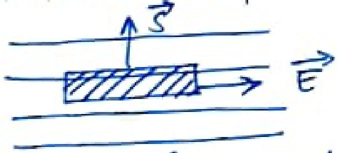
(a) For a flat surface : $\vec{E} \parallel \vec{s}$, $\vec{E}$ constant over the surface.

$\theta = 0°$, $\phi = E.s$

if $\vec{E} \nparallel \vec{s} \Rightarrow \phi = \vec{E}.\vec{s}$



(b) For a surface parallel to the field :-



$\vec{E} \perp \vec{s}$, $\theta = 90° \Rightarrow \phi = 0$

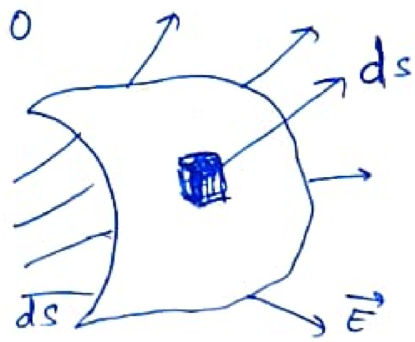
Non-Uniform Field :-

$\vec{E}$ - Not Uniform or $\vec{s}$ - Not flat

consider small area element, ds

Electric Flux, $d\phi$ through an area element, ds is defined by, $d\phi = \vec{E}.d\vec{s}$

$$\Rightarrow \phi = \int_s \vec{E}.d\vec{s} = \int_s E\,ds\cos\theta$$



Closed Surface :- Lines going out = +ve

lines going in = −ve

$$\phi_E = \oint \vec{E}.d\vec{s}$$

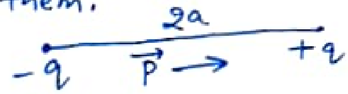
ELECTRIC DIPOLE:- An Electric Dipole is a pair of equal & opposite point charges, q and $-q$ separated by a distance, $2a$.

Ideal dipole: $2a \to 0$ **Point Dipole**: $2a \to 0$ & $q \to \infty$: p-finite

$-q \xrightarrow{2a} +q$

Dipole Moment:- ($\vec{p}$) It's a measure of the strength of electric dipole. It is a vector quantity whose magnitude is equal to product of the magnitude of charge and distance b/w them.

$$\vec{p} = q(2\vec{a}) \quad \text{OR} \quad |\vec{p}| = q(2a)$$



Unit: (C-m) **Direct^n**: from negative charge to positive charge.

Electret:- Permanent dipole, $\vec{p}$: independent of $\vec{E}$

Molecular Dipoles :- Net q on a molecule $= 0$, +ve & -ve ions don't completely overlap in most molecules ∴ form dipole.

if not, $\vec{p}$ is induced by $\vec{E}_{ext}$

www.physicsinduction.com

Polarization

Reason:- $\vec{E}_{ext}$, Hetero-nuclear bond, excess +ve on one & -ve on another.

3 types of Dipoles –
- Permanent
- Instantaneous : chance, temporary
- Induced

$p = 6.2 \times 10^{-30}$ C-m

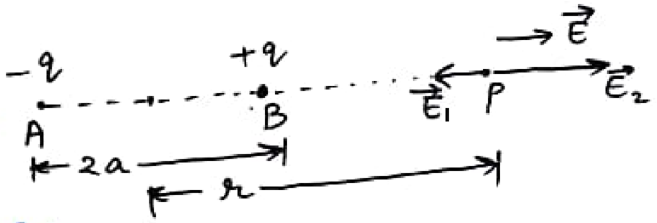
$\vec{p}$ of H_2O:-

DIPOLE FIELD:- It's the space around the dipole in which its electric effects can be experienced.

(i) The Electric Field Intensity on Axial line of an Electric Dipole:-

Electric field intensity at P:

- due to charge, $-q$: $|\vec{E}_1| = \dfrac{Kq}{(r+a)^2}$
- due to charge, $+q$: $|\vec{E}_2| = \dfrac{K\,q}{(r-a)^2}$



∴ **Net Electric field intensity at P :**

$$|\vec{E}| = |\vec{E}_2| - |\vec{E}_1| = Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = Kq \cdot \frac{4ar}{(r^2-a^2)^2}$$

$$\Rightarrow \vec{E} = K\frac{q \times 2a \times 2r}{(r^2-a^2)^2} = \boxed{K|\vec{p}| \cdot \frac{2r}{(r^2-a^2)^2}}$$

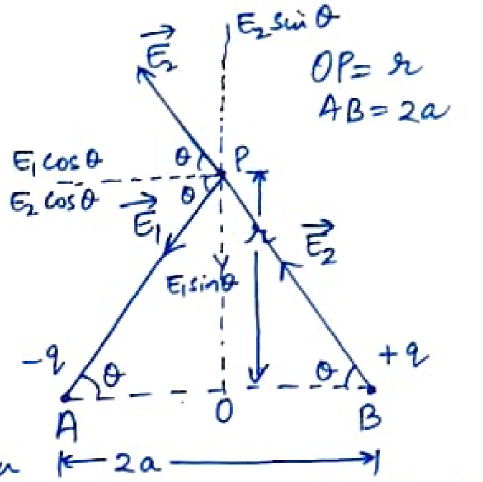
for $r >> a$, $|\vec{E}| = \dfrac{2K|\vec{p}|}{r^3}$

www.physicsinduction.com

(ii) The Electric Field Intensity on Equitorial line of an Electric Dipole :

The Electric Field Intensity at Point, P?

$AP = BP = \sqrt{r^2+a^2}$



Electric Field Intensity at P due to $-q$:

$$|\vec{E_1}| = \frac{Kq}{(r^2+a^2)} \quad \text{--- (1)}$$

Electric field Intensity at P due to $+q$:

$$|\vec{E_2}| = \frac{Kq}{(r^2+a^2)} \quad \text{--- (2)}$$

From (1) & (2), $|\vec{E_1}| = |\vec{E_2}|$

On resolving $\vec{E_1}$ & $\vec{E_2}$ into components, we can see that $|\vec{E_1}|\sin\theta$ & $|\vec{E_2}|\sin\theta$ point in opposite directions & cancel out each other.

$\therefore$ Resultant Intensity at P,

$$|\vec{E}| = |\vec{E_1}|\cos\theta + |\vec{E_2}|\cos\theta$$

$$= 2|\vec{E_1}|\cos\theta \quad \{\because |\vec{E_1}| = |\vec{E_2}|\}$$

$$= 2\left(\frac{Kq}{r^2+a^2}\right)\frac{a}{\sqrt{r^2+a^2}} = \frac{KP}{(r^2+a^2)^{3/2}}$$

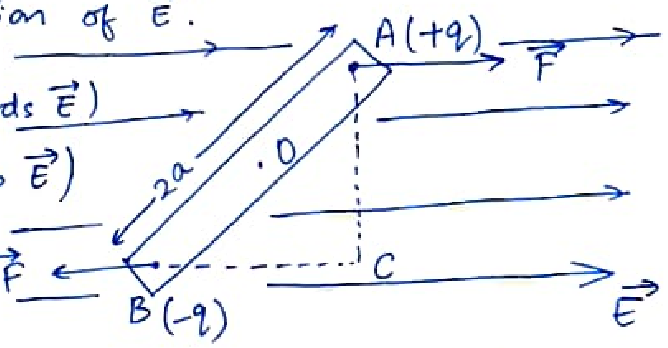
for $r >> a$

$$|\vec{E}| = \frac{KP}{r^3}$$

In Vector form: $\vec{E} = \frac{-K\vec{P}}{(r^2+a^2)^{3/2}}$

(iii) Dipole in a Uniform External Field:-

Consider an Electric Dipole, in a uniform Electric Field, $\vec{E}$ at an angle, θ with the direction of $\vec{E}$.



Force on $+q$ at A $= q\vec{E}$ (towards $\vec{E}$)

Force on $-q$ at B $= q\vec{E}$ (opp. to $\vec{E}$)

$|\vec{F_1}| = |\vec{F_2}|$ ($F_{net} = 0$)

These forces being equal, unlike & parallel, form a couple, which rotates the dipole in clockwise directn.

Draw $AC \perp \vec{E}$ & $BC \parallel \vec{E}$

$\therefore \perp$r dist. b/w the forces = Arm of couple = AC

As, Torque = moment of the couple

= Force × Arm of couple

$\vec{\tau} = \vec{F}\times\vec{AC} = F \times AB\sin\theta = qE\times 2a\sin\theta = PE\sin\theta$ ($\because P = q\times 2a$)

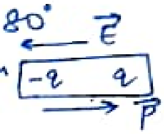
$$\boxed{\vec{\tau} = PE\sin\theta = \vec{P}\times\vec{E}}$$

$\vec{\tau}$ is $\perp$r to $\vec{P}$ & $\vec{E}$

Sp. cases :-

- when $\vec{P}$ is aligned to $\vec{E}$, $\theta = 0°$ $\Rightarrow \tau = PE\sin 0° = 0 \Rightarrow$ stable Equilibrium
- when dipole is held in a direct^n. opp. of $\vec{E}$, $\theta = 180°$

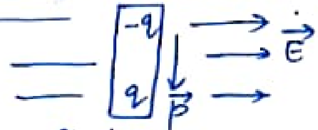
$\Rightarrow \tau = PE\sin 180° = 0 \Rightarrow$ Unstable equilibrium



- $\vec{P} \perp \vec{E}$: $\theta = 90°$

$\Rightarrow \tau = PE\sin 90° = PE$

$\rightarrow$ Max^m torque



- When $\vec{E}$ is not uniform : $F_{net} \neq 0$, $\tau \neq 0$

2 cases –
- (-q) —— (+q) $\rightarrow \vec{E}$, $\rightarrow \vec{P}$: $F_{net} \rightarrow$ towards increasing field
- (-q) —— (+q) $\leftarrow \vec{E}$: $F_{net} \rightarrow$ towards decreasing field.

(iv) P.E. of Dipole in a Uniform $\vec{E}$:- P.E $\rightarrow$ Energy possessed by the dipole by virtue of its pos^n

Small amt of W.D. in rotating the dipole thro' a small angle, $d\theta$ against the torque: $dW = \tau d\theta = PE\sin\theta\, d\theta$

$\therefore$ Total W.D. in rotating the dipole from θ_1 to $\theta_2 = W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$

$$W = \int_{\theta_1}^{\theta_2} PE\sin\theta\, d\theta = -PE(\cos\theta)_{\theta_1}^{\theta_2} = -PE(\cos\theta_2 - \cos\theta_1)$$

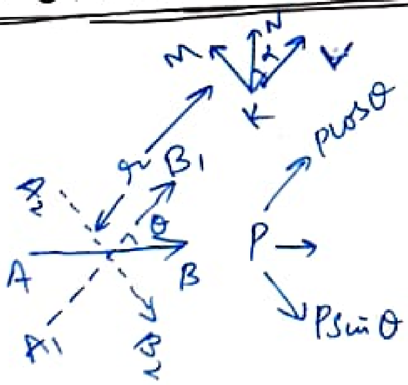
Suppose, dipole is at rt $\angle$ to $\vec{E}$, initially $\therefore \theta_1 = 90°$, set it to θ later.

$$\Rightarrow W = -PE(\cos\theta - \cos 90°) = -PE\cos\theta$$

$$\therefore U(P.E.) = -PE\cos\theta = -\underbrace{\vec{P}\cdot\vec{E}}_{\text{scalar}} \quad (\text{W.D. stored as P.E.})$$

$$\boxed{U = W = -\vec{P}\cdot\vec{E}}$$

(v) Electric Field due to short Electric Dipole :- Neglect a



$$|\vec{E_1}|_{\text{axial line}} = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3} \quad (\text{along } KL)$$

$$|\vec{E_2}|_{\text{equi. line}} = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3} \quad (\text{along } KM)$$

$$|\vec{E}|_{\text{along } KN} = \sqrt{E_1^2 + E_2^2}$$

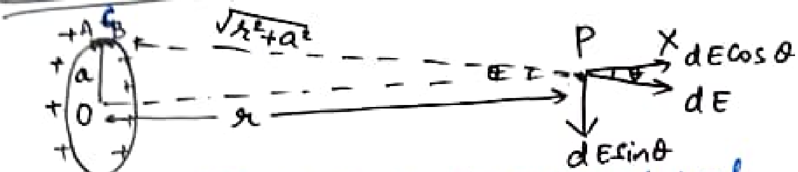
$$|\vec{E}| = \sqrt{\left(\frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{P\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$|\vec{E}| = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$|\vec{E}| = \frac{KP}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$\tan\alpha = \frac{LN}{KL} = \frac{KM}{KL} = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3} \times \frac{4\pi\varepsilon_0 r^3}{2P\cos\theta} = \frac{1}{2}\tan\theta$$

(vi) Electric Field Intensity at any point on the axis of a uniformly charged Ring.



Consider a circular loop of wire of negligible thickness, radius, a & centre, O. Let the loop carry a total charge, q distributed uniformly over its circumference.

To find: $\vec{E}$ at P due to charged loop

Consider a small element, AB of the loop.

$AB = dl$

Charge on the element, $AB = dq = \frac{q}{2\pi a} \cdot dl$

$|d\vec{E}|$ due to dq at P $= \frac{1}{4\pi\epsilon_0} \times \frac{dq}{PC^2} = \frac{K\,dq}{(r^2+a^2)}$

$d\vec{E}$ can be resolved into two rect. comp: $dE\cos\theta$ & $dE\sin\theta$.

As, loop can be considered to be made up of a large no. of pairs of diametrically opp. elements $\therefore \sum dE\sin\theta = 0$

Hence, Resultant $|\vec{E}| = \sum dE\cos\theta$

$$= \sum \frac{K\,dq}{(r^2+a^2)} \cdot \frac{r}{\sqrt{r^2+a^2}}$$

$$= \frac{Kr}{(r^2+a^2)^{3/2}} \sum dq = \frac{Kr}{(r^2+a^2)^{3/2}} \sum \frac{q}{2\pi a} \cdot dl$$

$$= \frac{Kr}{(r^2+a^2)^{3/2}} \times \frac{q}{2\pi a} \sum dl = \frac{Kr}{(r^2+a^2)^{3/2}} \times \frac{q}{\cancel{2\pi a}} \times \cancel{2\pi a}$$

$$\Rightarrow \boxed{|\vec{E}| = \frac{Kqr}{(r^2+a^2)^{3/2}}}$$

- When P lies at the centre, $r = 0 \Rightarrow \vec{E} = 0$
- When $r >> a$, neglect $a^2 \Rightarrow |\vec{E}| = \frac{Kqr}{r^3} = \underbrace{\frac{Kq}{r^2}}_{\text{behaves as a pt. q}}$ along PX

DISTRIBUTION OF CHARGES:-

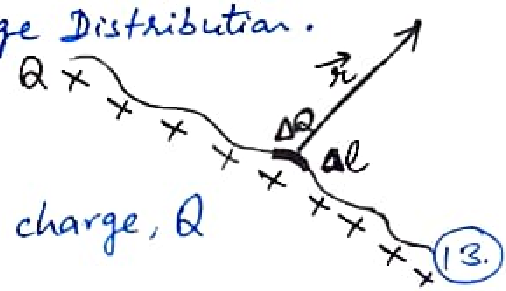
Charge distribution is of two types: (a) **discrete** :- Distribution of charges with considerable space in b/w the charges.

(b) **continuous** :- Distribution of charges with negligible space in b/w the charges.

CONTINUOUS CHARGE DISTRIBUTION:-

A system of closely spaced charges is said to form Continuous Charge Distribution.

(i) Linear Charge Distribution:-



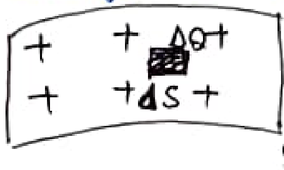
λ: Linear Charge Density

Consider a wire of length, l; having a charge, Q

Let ΔQ is the charge contained in the line element, Δl.
The linear charge density, λ of a wire is defined by:

$$\boxed{\lambda = \frac{\Delta Q}{\Delta l}}$$

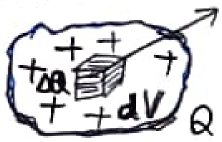
(ii) Surface Charge Distribution:- σ: Surface charge density



Let the charge, Q is distributed over the surface, S. Consider an area element, Δs & specify the charge, ΔQ on that surface.
We, then define surface charge density, σ at the area element by,

$$\boxed{\sigma = \frac{\Delta Q}{\Delta S}}$$

(iii) Volume Charge Distribution:- ρ: Volume Charge Density
Consider a continuous charge distribution in space. Let the charge, Q be distributed over volume, V. A small volume element ΔV occupies the charge, ΔQ. $\therefore$ $\boxed{\rho = \frac{\Delta Q}{\Delta V}}$



GAUSS'S LAW:- Total Electric Flux, over a closed surface, S in vacuum is $1/\varepsilon_0$ times the total charge contained inside, S.

i.e., $\phi_E = \oint \vec{E} \cdot d\vec{s} = \frac{q_{enclosed}}{\varepsilon_0}$

- Electric Field, $\vec{E}$ is the resultant electric field due to all the charges existing in the space.
- $q_{enclosed}$ includes only those charges, which are inside the cld surface.

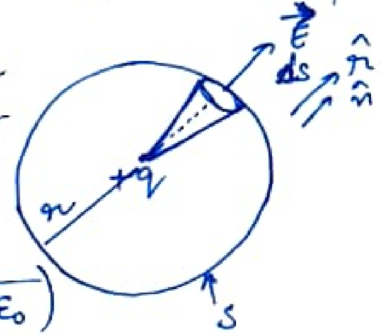
Strategy for solving Guass's Law Problems:-

→ To apply Guass's law, we construct a "Guassian surface" enclosing the charge.
→ Select a Guassian Surface with symmetry that matches the charge distribution.
→ Draw the Guassian surface so that the Electric Field is either zero or constant at all points on the Guassian surface.
→ Use symmetry to determine the direct^n of $\vec{E}$ on the Guassian surface.
→ Evaluate the surface integral, ϕ_E
→ Determine the charge inside the Guassian surface.
→ Solve for $\vec{E}$.

Note:- Take care, not to let Guassian surface pass thro' any discrete charge $\because$ $\vec{E}$ due to discrete charges isn't well defined at the location of any charge.

• Guass's law is based on inverse square dependence on distance contained in the coulomb's law. Any violation of Guass's law will indicate departure from the inverse square law.

Proof of Guass's law :- Consider a Guassian surface in the form of a sphere at radius, r for charge, $+q$.



As, $\vec{E} = \frac{Kq}{r^2}\hat{r}$, $d\vec{s} = ds.\hat{n}$ $\left(K = \frac{1}{4\pi\varepsilon_0}\right)$

$$\therefore \vec{E}.d\vec{s} = \frac{Kq}{r^2}\hat{r}.ds.\hat{n}$$

$$= \frac{Kq}{r^2}.ds \quad (\because \hat{r}.\hat{n} = 1)$$

$$\Rightarrow \oint \vec{E}.d\vec{s} = \frac{Kq}{r^2}\oint ds = \frac{Kq}{r^2}(4\pi r^2) = \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2) = \frac{q}{\varepsilon_0}$$

Total Electric Flux for a system consisting of charges $(q_1, q_2 \ldots q_n)$ is

$$\phi_E = \phi_{E_1} + \phi_{E_2} + \phi_{E_3} + \cdots + \phi_{E_n}$$

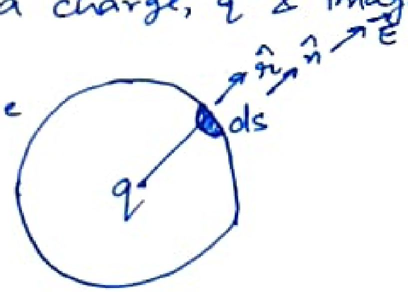
$$= \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \cdots + \frac{q_n}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \cdots + q_n) = \frac{q}{\varepsilon_0}$$

Coulomb's Law from Guass's Law :- Consider a charge, q & imagine spherical Guassian surface.

mag. of $\vec{E}$ = const everywhere on the surface

direct^n of $\vec{E}$ = radially outwards.



Consider a small area element, ds on the surface.

As, $\oint_s \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$

$$\Rightarrow \oint_s E\,ds\cos\theta = \frac{q}{\varepsilon_0}$$

www.physicsinduction.com

$$\Rightarrow E\oint ds = \frac{q}{\varepsilon_0} \quad (\because \theta = 0^\circ)$$

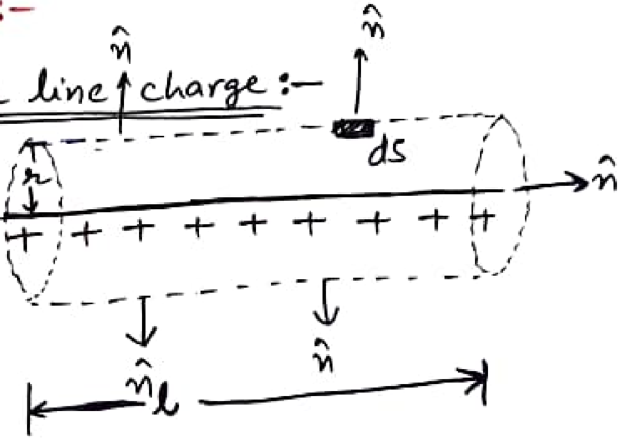
$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

If another charge, q_0 is placed at this pt. then, Force on q_0 would be

$$\boxed{F = q_0 E = \frac{q q_0}{4\pi\varepsilon_0 r^2}} \leftarrow \text{Coulomb's law}$$

APPLICATIONS OF GAUSS'S LAW :-

(i) Electric Field Intensity due to a line charge :-



$$\phi_E = \oint_s \vec{E}\cdot d\vec{s} = \oint_s \vec{E}\, ds\, \hat{n}$$

$$= E\oint_s ds = E(2\pi r l) \quad \text{— (1)}$$

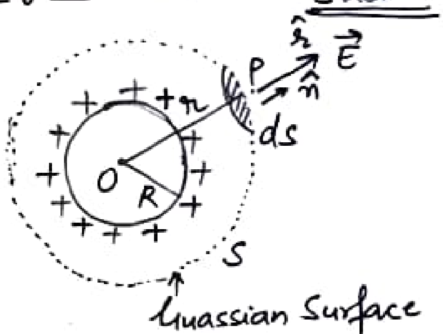
$$\phi_E = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \quad \text{— (2)}$$

From (1) & (2) : $E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} \qquad E \propto \frac{1}{r}$$

(ii) Electric Field Intensity due to a uniformly charged spherical shell :-

σ : Uniform surface charge density



(a) Field outside the shell :-

$$\oint_s \vec{E}\cdot d\vec{s} = \oint \vec{E}\cdot\hat{n}\, ds = \frac{q}{\varepsilon_0}$$

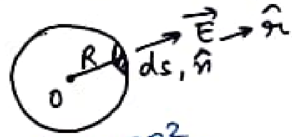
$$\Rightarrow E\oint ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

$\vec{E}$: directed outward if $q > 0$
$\vec{E}$: directed inward if $q < 0$

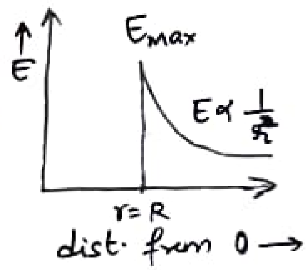
(b) At a point on the surface of the shell :-

$r = R$



As, $E\oint ds = \frac{q}{\varepsilon_0} \Rightarrow E(4\pi R^2) = \frac{q}{\varepsilon_0} = \frac{4\pi R^2\sigma}{\varepsilon_0}$ $\qquad q = 4\pi R^2\sigma$

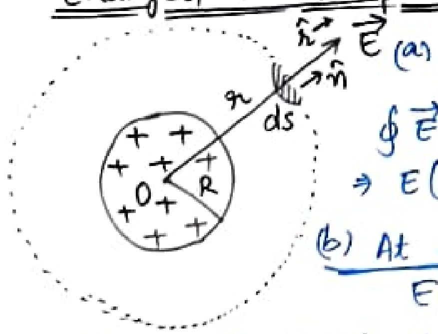
$$\Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$



(c) Inside the spherical shell :- $q = 0 \therefore E = 0$

(iii) Electric Field Intensity due to a non-conducting charged solid sphere :-

ρ : Volume charge density
σ : Surface charge density



(a) Field outside the sphere :-

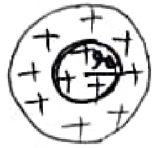
$$\oint \vec{E}\cdot d\vec{s} = \oint \vec{E}\,\hat{n}\, ds = E\oint ds = E(4\pi r^2)$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

(b) At a point on the surface of the sphere — $r = R$

$$E(4\pi R^2) = \frac{q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$$

(c) At a pt inside the sphere $r < R$:



If q' is the charge enclosed by the sphere, S, then;

$$\oint_s \vec{E}\cdot d\vec{s} = \oint_s \vec{E}\,\hat{n}\, ds = E\oint_s ds = \frac{q'}{\varepsilon}$$

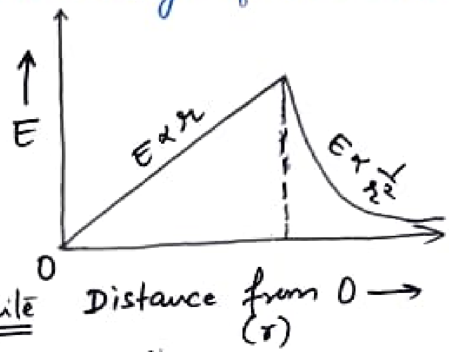
$$E(4\pi r^2) = \frac{q'}{\varepsilon}$$

$$\Rightarrow E = \frac{q'}{4\pi\varepsilon r^2}$$

charge inside S, q' = volume of S × volume density of the charge

$$\Rightarrow q' = \frac{4}{3}\pi r^3 \rho$$

$$\Rightarrow E = \frac{4\pi r^3 \rho}{3 \times 4\pi\varepsilon r^2} \Rightarrow \boxed{E = \frac{r\rho}{3\varepsilon}}$$



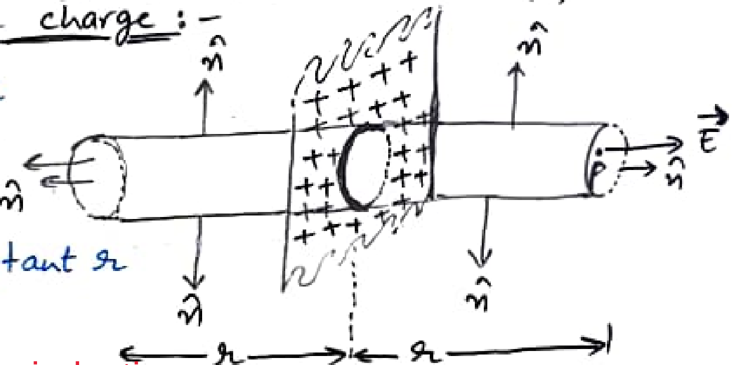
(d) At the centre, $r=0$:- $\boxed{E=0}$ ∵ $r=0$

(iv) Electric Field Intensity due to a thin infinite plane sheet of charge :-

consider a thin, infinite plane sheet of charge.

σ: surface charge density

we have to calculate Electric field intensity at any pt., P, distant r from the sheet.



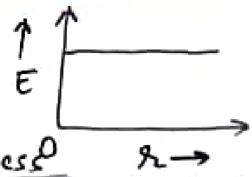
At the edges : $\vec{E} \parallel \hat{n}$ www.physicsinduction.com

$$\phi_E(\text{edges}) = 2\vec{E}\cdot\hat{n}\,ds = 2E\,ds$$

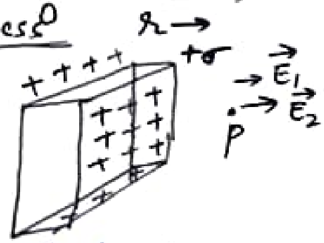
On the curved surface of the cylinder: $\vec{E} \perp \hat{n}$. No contribution to Electric Flux is made by the curved surface of the cylinder. i.e., $\phi_E = 0$

∴ Total Electric Flux over the entire surface of the cylinder $= 2E\,ds$

$$\underset{(\text{Entire Surface})}{\phi_E} = 2E\,ds = \frac{q}{\varepsilon_0} = \frac{\sigma ds}{\varepsilon_0} \quad \therefore \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$



★ If the infinite plane sheet has uniform thickness



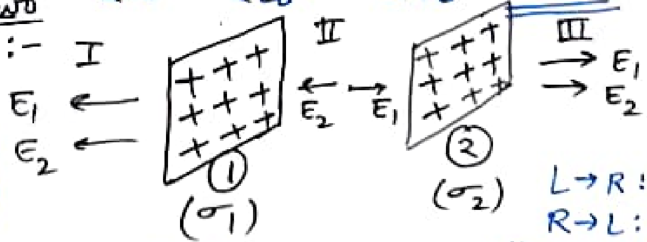
Electric Field at any pt., P due to each surface :

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

∴ Net Electric Field Intensity at P, $E = E_1 + E_2$ (superposition Principle)

$$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

★ Electric Field Intensity due to two thin infinite parallel sheets of charge :-



Region-I : $E_I = -E_1 - E_2 = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$

$$= \frac{-1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

L→R : +ve
R→L : −ve
(by convention)

Let $\sigma_1 > \sigma_2 > 0$

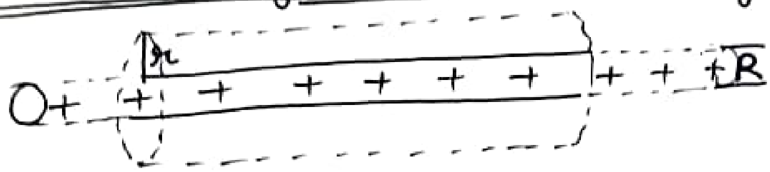
Region-II :- $E_{II} = E_1 - E_2$

$$= \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$$

Region-III

$$E_{III} = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

(v) Electric Field Intensity due to a conducting cylinder (Infinite)



$\underline{r>R}$: $\phi_E = E(2\pi r l)$

Also, $\phi_E = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$ $\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$

$\underline{r=R}$: $\phi_E = \frac{\lambda}{2\pi\varepsilon_0 R}$, $E \propto \frac{1}{R}$

$\underline{r<R}$: $q=0 \Rightarrow E=0$

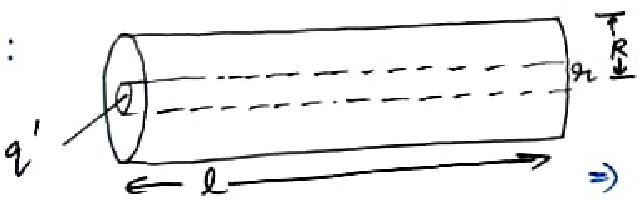
(vi) Electric Field Intensity due to a non-conducting cylinder :-

$\underline{r>R}$: $\phi_E = \oint \vec{E}\cdot d\vec{s} = E(2\pi r l)$

Also, $\phi_E = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$, $E \propto \frac{1}{r}$

$\underline{r=R}$: $E = \frac{\lambda}{2\pi\varepsilon_0 R}$, $E \propto \frac{1}{R}$

$\underline{r<R}$:



$\phi_E = E(2\pi r l)$

& $\phi_E = \frac{q'}{\varepsilon} = \frac{\pi r^2 l \rho}{\varepsilon}$ $\left[\rho = \frac{q'}{V} = \frac{q'}{\pi r^2 l}\right]$

$\Rightarrow E = \frac{\pi r^2 l \rho}{2\pi r \cdot \varepsilon l} = \underline{\underline{\frac{r\rho}{2\varepsilon}}}$, $E \propto r$

$\underline{r=0}$: $E=0$