SHORT NOTES: CLASS 11

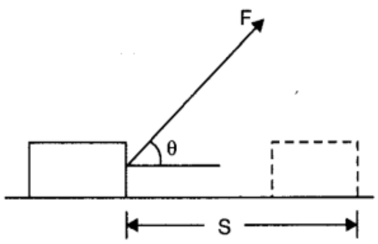
CHAPTER 5: WORK, ENERGY AND POWER

WORK: Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of the applied force. http://www.physicsinduction.com

It is measured by the product of the force and the distance moved in the direction of the force,
i.e., W = F.s

• If an object undergoes a displacement 's' along a straight line while acting on a force F that makes an angle θ with s. The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement.

i.e. $W = Fs\cos\theta = \vec{F}\cdot\vec{s}$



- Work done is a scalar quantity and is measured in Joule, J.
- dim[W] = $[M^1L^2T^{-2}]$ www.physicsinduction.com
- W = 0 if F ⊥ S i.e., if $\theta = 90^o$
- W = positive, if the angle b/w F and s is acute, $0^0 < \theta < 90^o$
- W = negative, if the angle b/w F and s is obtuse, $90^0 < \theta < 180^o$

Work done by a variable force:

To calculate the work done in moving the body from one to another under the action of variable force, We assume the displacement to be made up of a large number of infinitesimal small displacements. For the small displacement Δs, we can take F(s) as approximately constant.

$$\Delta W = F(s)\,\Delta s$$

$$\text{Total W.D.} = W = \sum \Delta W = \sum F(s)\,\Delta s$$

$$W = \sum F(s)\,\Delta s \text{ (if } \Delta s \quad 0)$$

In terms of integral calculus, If the force applied varies with time/position, the work done is given by:
$W = \int \vec{F}\cdot d\vec{s}$

1Joule: One Joule is equivalent to one Newton of force causing a displacement of one meter.

1 Joule = 1 Newton * 1 meter   1 J = 1 N * m

- If we plot a graph between the force applied and the displacement, then work done can be obtained by finding the area under the F-s graph.

Wok done by a spring:

If a spring is stretched or compressed by a small distance from its unstretched configuration, the spring will exert a force on the block given by: F= -kx

where x is compression or elongation in spring, www.physicsinduction.com

k is the spring constant whose value depends inversely on the unstretched length and the nature of the material of the spring. The negative sign indicates that the direction of the spring force is opposite to x, the displacement of the free-end.

Work done by a spring when the block is displaced by x :

$W = \int_0^x \vec{F} \quad d\vec{x}$ $W = \int F\,dx\cos 180^0$ $W = -\int_0^x F\,dx$

$W = -\int_0^x kx\,dx = -\frac{1}{2}kx^2$

Work done by an agent in giving an elongation or compression of x is given by $\frac{1}{2}kx^2$.

Work done by a spring when the block is displaced from x_1 to x_2 : $W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

CONSERVATIVE AND NON-CONSERVATIVE FORCES:

1. **Conservative forces:** A force is said to be conservative if the work done in moving an object from

one point to another depends only on the initial and final positions of the object and does not depend upon the path followed.

Characteristics of a conservative force.

- It depends on the initial and final position irrespective of the path taken.
- In any closed path, the work done by a conservative force is zero.

Examples of conservative forces are:

(i) Gravitational force, (ii) Electrostatic force, (iii) magnetic force, (iv) Spring force

2. **Non-Conservative forces:** A force is said to be non-conservative if the work done in moving from one point to another depends upon the path followed.

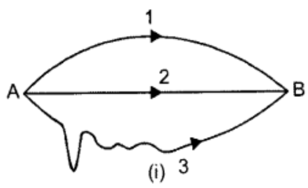
Characteristics of non-conservative force.

- It is path-dependent therefore, it also depends on the initial and final velocity.
- In any closed path, the total work done by a non-conservative force is not zero.

Examples of non-conservative forces are:

(i) Force of friction (ii) Viscous force

- Work done by friction in a round trip = negative
- Let W_1 be the work done in moving from A to B following path 1. W_2 through the path 2 and W_3 through the path 3.
- For a non-conservative force: $W_1 \neq W_2 \neq W_3$



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The law of conservation of energy holds goods for both conservative and non-conservative forces.

ENERGY: The energy of a body is its capacity to do work.

Units: Joule, ergs, calorie, KWh, eV

Dimensions: $[M^1L^2T^{-2}]$

KINETIC ENERGY: The energy possessed by a body by virtue of its motion is known as its kinetic energy. For an object of mass m and having a velocity v, the kinetic energy is given by:

$$K.E. \text{ or } K = \frac{1}{2} mv^2$$

K.E.= W.D. in moving a body from rest OR W.D. in stopping a moving body.

Let ds be the small displacement in the direction of F.

$dW = \vec{F} \cdot d\vec{s}$ = FdsCos0^0 = Fds

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$dW = m\frac{dv}{dt} \times ds = m\left(\frac{ds}{dt}\right) \times dv = mv\,dv$ www.physicsinduction.com

Total W.D., $W = \int_0^v mv\,dv = m\int_0^v v\,dV = \frac{mv^2}{2} = \frac{1}{2}mv^2$

K.E. is always positive. It depends upon the frame of reference chosen.

K.E. and Momentum:

P = mv, www.physicsinduction.com

K.E. = ½ mv^2 = $\frac{(mv)^2}{2m} = \frac{p^2}{2m}$

If P = constant, K.E. $\alpha \frac{1}{m}$

If m = constant, $P^2 \alpha$ K.E. => P $\alpha \sqrt{K.E.}$

If K.E. = constant, $P^2 \alpha$ m => P $\alpha \sqrt{m}$

WORK-ENERGY THEOREM:

According to the work-energy theorem, the work done by a force on a body is equal to the change in kinetic energy of the body. It is valid even in non-conservative forces.

- W.D. = Change in K.E. of a body = Δ(K.E.)
- Δ(K.E.) = difference between the final and initial Kinetic energies of the body.

- Work and kinetic energy are equivalent terms.
- Decrease in kinetic energy = W.D. against force => Retarding force

Consider an object of mass m moving with some initial velocity, u. let F be the force applied in the direction of displacement, s covered in time t, and suppose the velocity changes to v during this time interval.

A small amount of work done by F in the direction of motion:

$dW = \vec{F} \cdot d\vec{s} = Fds\cos 0^0 = F\,ds$

$$dW = F\,ds = ma\,ds = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{ds}{dt}\right)dv = mvdv$$

$W = \int_u^v mv\,dv = m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ = change in kinetic energy

$$\vec{F} = 0 \Rightarrow v = Constant \Rightarrow K.E. = Same$$

$$Resultant\ \vec{F} \perp v \Rightarrow speed = constant \Rightarrow K.E. = constant$$

K.E. changes when speed changes and it happens only when the resultant force has tangential component.

POTENTIAL ENERGY www.physicsinduction.com

The energy possessed by a body by virtue of its position or configuration is known as its potential energy. There are two common forms of potential energy: gravitational and elastic

Gravitational potential energy: Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

P.E. = V(h) = mgh

F= -mg (the -ve sign indicates that the gravitational force is downward.)

It is easily seen that F is the negative of the derivative of V(h), w.r.t. h.

$$F = -\frac{dV(h)}{dh} = -mg$$

P.E., V(x) is defined as: F(x) = -dV/dx (for 1-D motion)

The change in P.E. for a conservative force, ΔV is equal to the negative of the W.D. by the force.

Elastic potential energy: When an elastic body is displaced from its equilibrium position, work needs to be done against the restoring elastic force. The work done is stored up in the body in the form of its elastic potential energy.

If an elastic spring is stretched (or compressed) by a distance, x from its equilibrium position, then its elastic potential energy is given by : V= ½ kx^2 www.physicsinduction.com

where, k —> force constant of the given spring, depends upon the length, radius, and nature of the spring.

k is large for stiff spring & small for soft spring.

By Hook's law: Restoring force α stretch/compression

⇨ -F α x ⇨ -F = kx ⇨ F=-kx

Small amount of work done in increasing the length of the spring by dx:

dW = - Fdx = kxdx

$W = \int_{x=0}^{x=x} kx\,dx$

$$W = k\left(\frac{x^2}{2}\right)_0^x = \frac{1}{2}kx^2$$

Electric potential energy: It is associated with the state of separation of charged particles that interact electrically.

THE CONSERVATION OF MECHANICAL ENERGY: The total mechanical energy of the system is conserved if the forces doing work on it are conservative. M.E. is not conserved in the presence of friction. www.physicsinduction.com

M.E. = E = K + V(x)

K : always +ve and V: may be +ve/-ve

K = E – V ≥ 0

⇨ E ≥ V

- -ve value of E indicates bound state e.g. electron in an atom, satellite around the planet.
- M.E. is not conserved in the presence of friction.
- M.E. conservation occurs even when acceleration is constant or the acceleration varies.

THE LAW OF CONSERVATION OF ENERGY: The total energy of an isolated system remains constant.

- Energy may be transformed from one form to another
- Energy can neither be created, nor destroyed.
- Internal forces are conservative and external forces do no work.

Consider a situation where an object is thrown to a certain height, at different positions of the object, we will find total energy of the system. www.physicsinduction.com

At A: K.E. = 0, P.E. = mgh therefore, T.E. = mgh (total energy is entirely potential)

At C : $v^2 - u^2 = 2gh$ => $v^2 = 2gh$ (total energy is entirely kinetic)

K.E. = ½ mv^2 = ½ m (2gh) = mgh

P.E. = 0

=> T.E. = mgh

At B: let v' be the velocity of the object at a distance x from the height.

K.E. = ½ mv'^2, $v'^2 = 2gx$ => K.E. = mgx

P.E. = mg (h – x)

T.E = K.E. + P.E. = mgh

EQUIVALENCE OF MASS AND ENERGY:

According to Einstein, mass and energy are inter-convertible. That is, mass can be converted into energy and energy can be converted into mass.

$E = mc^2$ where, $c = 3 \times 10^8$ m/s,

When a body moves with velocity, v comparable to the velocity of light, c, its mass is given by:

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where, m_o is the rest mass.

POWER: It is the rate of doing work i.e., work done per unit time.

P = $\frac{dW}{dt}$ = $\vec{F} \cdot \vec{v}$ = Fv Cosθ www.physicsinduction.com

Where θ is the angle between the force, F and velocity, v.

$W = \int P\, dt$ – Area under P-t curve

dW = $\vec{F} \cdot ds$ = $\vec{F} \cdot \vec{v}dt$ if force is perpendicular to velocity, then W = 0

- Power is a scalar quantity.
- Dimensional formula of power is $[M^1L^2T^{-3}]$
- S.I. unit is Watt, W, Commercial unit is horse power, 1 h.p. = 746 W

COLLISIONS:

Collision is defined as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time.

Two key rules: (i) T.E. is conserved, (ii) conservation of linear momentum.

Two types: (i) Elastic collisions (ii) Inelastic collisions

Elastic collision:
A collision between two particles or bodies is said to be elastic if both the linear momentum and the kinetic energy of the system remain conserved.
Example: Collisions between atomic particles, atoms, marble balls and billiard balls.
Inelastic collision:
A collision is said to be inelastic if the linear momentum of the system remains conserved but its kinetic energy is not conserved. www.physicsinduction.com
Example: When we drop a ball of wet putty on to the floor then the collision between ball and floor is an inelastic collision.
COEFFICIENT OF RESTITUTION OR COEFFICIENT OF RESILIENCE:
Coefficient of restitution is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.
It is represented by e.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

e=1 : perfectly elastic collision, e=0: perfectly inelastic collision
ELASTIC COLLISION IN 1-D:
consider two particles of masses m_1 and m_2 moving with velocities u_1 and u_2 respectively collide head on such that v_1 and v_2 be their respective velocoties after collision.

a) $e = 1 \Rightarrow v_2 - v_1 = u_1 - u_2$……………………………………………………………….(i)
b) P : Conserved => $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$…………………………………….(ii)
c) K.E. is conserved => $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ www.physicsinduction.com
   ⇨ $m_2(v_2^2 - u_2^2) = m_1(u_1^2 - v_1^2)$…………………………………………………(iii)
   (iii) ÷(ii) gives

$$v_2 + u_2 = u_1 + v_1$$

$v_2 - v_1 = u_1 - u_2$……………………………Proved (i)
Calculation of velocities v_1 and v_2 :
$v_2 = u_1 - u_2 + v_1$…………………………………………………………………from (i)
Substitute v_2 in (ii),

$$m_1v_1 + m_2(u_1 - u_2 + v_1) = m_1u_1 + m_2u_2$$
$$m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1 = m_1u_1 + m_2u_2$$
$$v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$$
$$v_1 = \frac{(m_1 - m_2)u_1}{(m, + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)}$$

Now, $v_1 = u_2 - u_1 + v_2$…………………………………………………… from (i)

$$m_1(u_2 - u_1 + v_2) + m_2v_2 = m_1u_1 + m_2u_2$$
$$m_1u_2 - m_1u_1 + m_1v_2 + m_2v_2 = m_1u_1 + m_2u_2$$
$$(m_1 + m_2)v_2 = (m_2 - m_1)u_2 + 2m_1u_1$$
$$v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1u_1}{m_1 + m_2}$$

Cases:
Case-1: if $m_1 >> m_2$, $m_2 \approx 0$ => $\frac{m_1-m_2}{m_1+m_2} \approx 1, \frac{2m_2}{m_1+m_2} = 0 \ \frac{2m_1}{m_1+m_2} \approx 2$
⇨ $V_1 = u_1$ and $v_2 = 2u_1 - u_2$
Case – 2: If $m_2 >> m_1$, $m_1 \approx 0$ => $\frac{m_1-m_2}{m_1+m_2} \approx -1 \ \frac{2m_2}{m_1+m_2} \approx 2 \ \frac{2m_1}{m_1+m_2} = 0$
Case - 3 : If $m_1 = m_2 = m$(say) www.physicsinduction.com

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⇨ $V_1 = u_2$ and $v_2 = u_1$

Case – 4: if $u_2 = 0$

$$v_1 = \frac{(m_1 - m_2)u_1}{(m, + m_2)}$$
$$v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

INELASTIC COLLISION IN 1-D:

For a perfectly inelastic collision, $u_2 = 0$, objects stick together after collision and move with the common velocity, v.

Momentum is conserved but there is a loss of K.E.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$m_1u_1 = (m_1 + m_2)v$$
$$v = \frac{m_1u_1}{m_1 + m_2}$$

As, $\left(\frac{m_1}{m_1+m_2}\right) < 1 \Rightarrow v < u_1$ www.physicsinduction.com

$$KE_1 = \frac{1}{2}m_1u_1^2$$
$$KE_2 = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(m, + m_2)\left(\frac{m_1u_1}{m_1 + m_2}\right)^2 = \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$

Loss of Kinetic Energy: $KE_1 – KE_2$

$$= \frac{1}{2}m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$
$$= \frac{m_1u_1^2(m_1+m_2) - m_1^2u_1^2}{2(m_1+m_2)}$$
$$= \frac{m_1m_2u_1^2}{2(m_1 + m_2)} = positive$$

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