PHYSICS INDUCTION
An institute of Science & Mathematics

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NOTES : CLASS XI : MOTION IN A PLANE (CH-4) : PHYSICS

VECTORS :- Vectors are those physical quantities, which have both the magnitude and direction. For example: Velocity, Acceleration, force, momentum etc.

A vector is represented by an arrowed line.

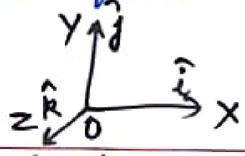
tail / Origin of the Vector (starting point) → ————————→ Tip / head of the Vector (Indicates Direction)

Length of the Vector indicates magnitude

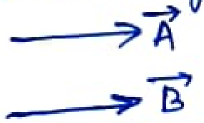
Magnitude of the vector is called its Absolute Value. $v = |\vec{v}|$

A FEW DEFINITIONS IN VECTOR ALGEBRA :-

(i) **Unit Vector :-** A Unit Vector of a given Vector is the vector of unit magnitude, which has the same direct^n as that of the given Vector. $\vec{A} = |\vec{A}|\,\underbrace{\hat{A}}_{\text{Unit Vector}} \Rightarrow \hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$



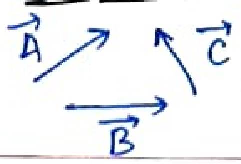
(ii) **Equal Vectors :-** Two Vectors are said to be equal if they have the equal magnitude & same direction.



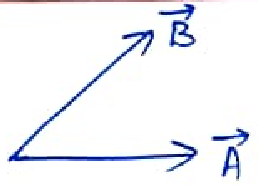
$|\vec{A}| = |\vec{B}|$

(iii) **Negative Vector :-** A negative Vector of a given Vector is a Vector of same magnitude but acting in opposite direction to the given Vector.

(iv) **Coplanar Vectors :-** Vectors acting in the same plane.



(v) **Coinitial Vectors :-** Initial point is common.



(vi) Collinear Vectors :- Vectors acting along the parallel straight lines.

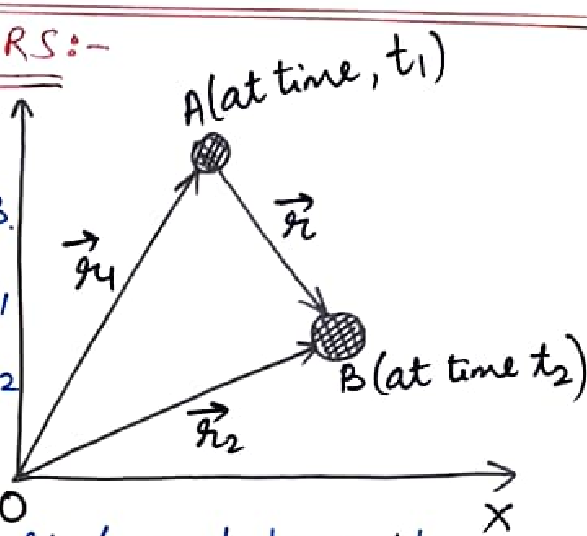
$\rightarrow \quad \leftarrow \quad \rightarrow \longrightarrow \quad \longrightarrow \quad \longleftarrow$

(vii) Zero/Null Vector :- Vector of zero magnitude & arbitrary direct^n. $\vec{0}$. $|\vec{0}| = 0$

$\vec{A} - \vec{A} = \vec{0}$,

$\vec{A} + \vec{0} = \vec{A}$, $\lambda\vec{0} = \vec{0}$, $\vec{0}\cdot\vec{A} = \vec{0}$

POSITION AND DISPLACEMENT VECTORS:-



Consider, an object moving in a plane from a point, A to a point B.

$\vec{OA} = \vec{r_1}$ = Position Vector of the object at t_1

$\vec{OB} = \vec{r_2}$ = Position Vector of the object at time, t_2

$\vec{r_1}$ & $\vec{r_2}$: straight line distance from O, direct^n of pos^n of particles w.r.t. O.

Displacement Vector :- $\vec{AB}$, straight line distance b/w initial & final positions of the object.

$$\boxed{\vec{AB} = \vec{r} = \vec{r_2} - \vec{r_1}}$$

MULTIPLICATION OF A VECTOR BY A REAL NO. / SCALAR :-

If $\lambda > 0$: $\lambda\vec{A} = \lambda|\vec{A}|.\hat{A}$ $\qquad \longrightarrow \vec{A}$ $\qquad \longrightarrow \lambda\vec{A}$

(Mag. is changed by a factor, λ & direct^n remains the same)

If $\lambda < 0$: $-\lambda\vec{A} = -\lambda|\vec{A}|.\hat{A}$ $\qquad \longrightarrow \vec{A}$ $\qquad \longleftarrow -\lambda\vec{A}$

(mag $\rightarrow$ $-\lambda$ times $\vec{A}$, direct^n is opp. to the direct^n of $\vec{A}$)

If λ: Scalar $\quad \vec{B} = \lambda\vec{A}$ $\quad$ e.g. $\vec{r} = \vec{v}.\underset{\text{scalar Quantity}}{t}$

(Another physical Quantity of diff. dimension, but same direct^n)

ADDITION OF VECTORS :-

$\vec{R}$: Resultant Vector : single Vector which prod. the same effect as is produced by ind. vectors together

(i) When the Vectors are acting in the same direct^n :

$\longrightarrow \vec{A}$ $\qquad$: $\qquad \xrightarrow{\vec{A}} \dashrightarrow^{\vec{B}}$

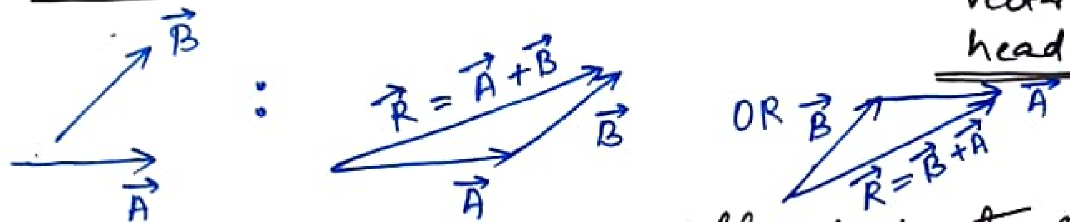
$\longrightarrow \vec{B}$ $\qquad\qquad \vec{R} = \vec{A} + \vec{B}$

(ii) When the Vectors are acting in the opposite directions :-

$\longrightarrow \vec{A}$ $\quad$: $\quad \longrightarrow \vec{A}$ $\quad$: $\quad \xrightarrow{\vec{A}} \dashrightarrow^{-\vec{B}}$

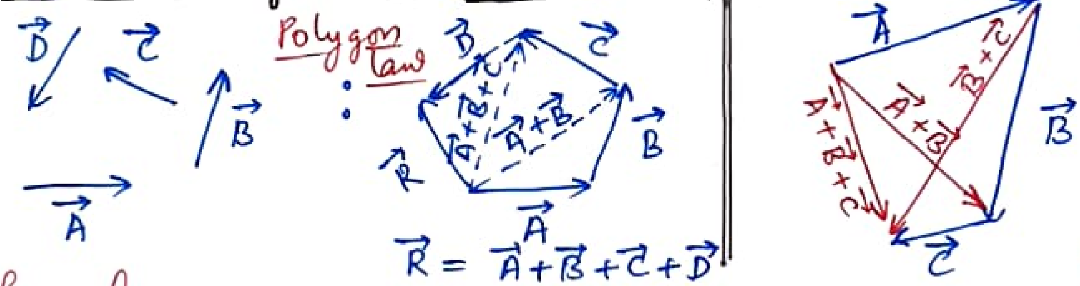
$\longleftarrow \vec{B}$ $\quad\quad \longrightarrow -\vec{B}$ $\quad\quad \vec{R} = \vec{A} - \vec{B}$

(iii) When two Vectors act at some angle :- (Triangle method of vector Addⁿ // head-to-tail Method)



If 2 vectors, acting on a part at the same time are represented by mag. & directⁿ by the 2 sides of a Δ taken in one order, then Resultant vector is represented in mag. & directⁿ by the 3rd side of the Δ taken in opp. order

(iv) When no. of vectors, act in different directions :-



Imp. facts of Vector Addition :

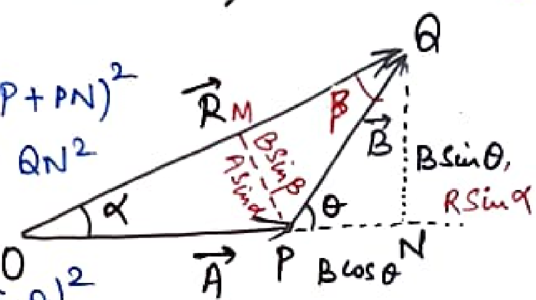
(i) Vectors of same nature alone can be added e.g. $\vec{r} = \vec{r}_1 + \vec{r}_2$

(ii) Vector Addⁿ is commutative :- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(iii) Vector Addⁿ is Associative :- $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

TRIANGLE LAW OF VECTORS ADDITION :- $\vec{R} = \vec{A} + \vec{B}$

(Magnitude And Direction of Resultant Vector, $\vec{R}$) Draw $QN \perp OP$ (extended)



Magnitude of R :-

$$OQ^2 = QN^2 + ON^2 = QN^2 + (OP + PN)^2$$

$$\Rightarrow OQ^2 = (OP^2 + PN^2 + 2 \cdot OP \cdot PN) + QN^2$$

$$\Rightarrow R^2 = A^2 + (B\cos\theta)^2 + 2AB\cos\theta + (B\sin\theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

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Direction of R :-

$$\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

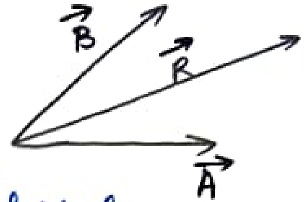
In ΔOQN : $QN = B\sin\theta = R\sin\alpha \Rightarrow \frac{R}{\sin\theta} = \frac{B}{\sin\alpha}$ — (1)

In ΔOPM : $PM = A\sin\alpha$; In ΔPQM; $PM = B\sin\beta$

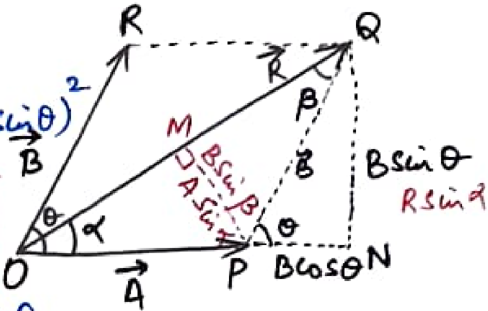
$\Rightarrow A\sin\alpha = B\sin\beta \Rightarrow \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$ — (2)

from (1) & (2) : $\boxed{\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}}$

PARALLELOGRAM LAW OF VECTORS ADDITION :-



If two vectors, acting on a particle at the same time, be represented by two adjacent sides of a parallelogram drawn from a point, their Resultant Vector is represented in mag. & direct^n by the diagonal of the llgm, drawn from the same pt. $\vec{R} = \vec{A} + \vec{B}$



Magnitude of R :-

$$\underset{R^2}{OQ^2} = ON^2 + QN^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$\Rightarrow \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

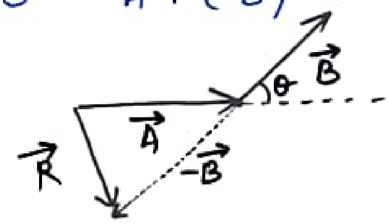
Direction of R :- $\tan\alpha = \dfrac{QN}{OP + PN} = \dfrac{B\sin\theta}{A + B\cos\theta}$

$B\sin\theta = R\sin\alpha$, $B\sin\beta = A\sin\alpha$ $\therefore$ $\boxed{\dfrac{R}{\sin\theta} = \dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}}$

SUBTRACTION OF VECTORS :- $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$\angle$ b/w $\vec{A}$ & $\vec{B}$: θ

$\therefore$ $\angle$ b/w $\vec{A}$ & $-\vec{B}$: $(180° - \theta)$



$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$$
$$= \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

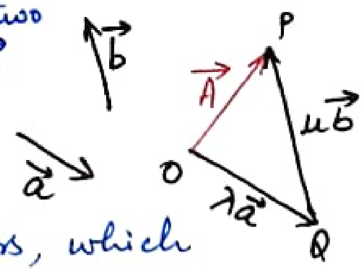
& $\tan\beta = \dfrac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)} = \dfrac{B\sin\theta}{A - B\cos\theta}$

* Vector subtraction doesn't follow commutative & Associative Laws.

$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$, $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

RESOLUTION OF VECTORS :-

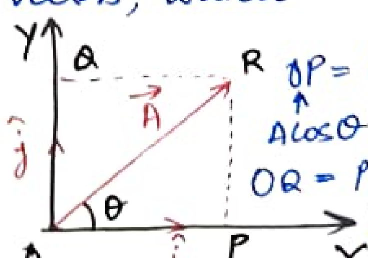
$\vec{a}$ & $\vec{b}$ be any two non-zero vectors with different directions & Let $\vec{A}$ be another vector in the same plane.



$\vec{OP} = \vec{OQ} + \vec{QP}$ $\Rightarrow$ $\boxed{\vec{A} = \lambda\vec{a} + \mu\vec{b}}$

(Splitting of a Vector into two or more vectors, which together produce the same effect)

Rectangular components :- (In 2D)



$OP = A_x$
$A\cos\theta$
$OQ = PR = A_y$
$A\sin\theta$

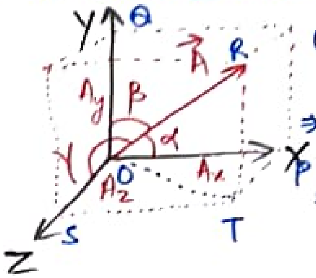
$$\vec{A} = \vec{A_x} + \vec{A_y} = A_x\hat{i} + A_y\hat{j}$$
$$= A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$
$$(\because A_x = A\cos\theta,\ A_y = A\sin\theta)$$

$$A_x^2 + A_y^2 = (A\cos\theta)^2 + (A\sin\theta)^2 = A^2$$

$$\Rightarrow \boxed{A = \sqrt{A_x^2 + A_y^2}}$$

$$\tan\theta = \frac{PR}{OP} = \frac{A_y}{A_x}$$

(In 3-D) Let α, β, γ are the angles which $\vec{A}$ makes with x, y and z axes respectively. then,



$OR^2 = OT^2 + RT^2$

$\Rightarrow OR^2 = OP^2 + OS^2 + TR^2$

$= OP^2 + OS^2 + OQ^2$

$$A_x = A\cos\alpha, \quad A_y = A\cos\beta, \quad A_z = A\cos\gamma$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

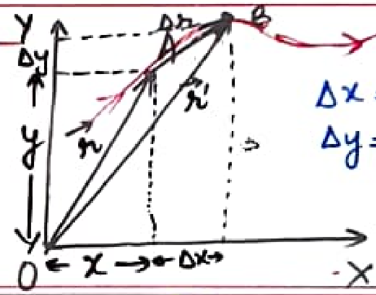
$$= A\cos\alpha\,\hat{i} + A\cos\beta\,\hat{j} + A\cos\gamma\,\hat{k}$$

$$\therefore \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\Rightarrow A = \sqrt{A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma} = A\sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$$

$$\Rightarrow \boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

POSITION VECTOR AND DISPLACEMENT :-



$$\Delta x = x' - x$$
$$\Delta y = y' - y$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r}' = x'\hat{i} + y'\hat{j} = (x+\Delta x)\hat{i} + (y+\Delta y)\hat{j}$$

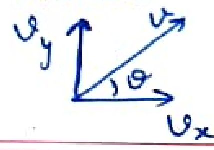
$$\Delta\vec{r} = \vec{r}' - \vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

VELOCITY :-

Avg. velocity, $\vec{v} = \dfrac{\text{displacement}, \Delta\vec{r}}{\text{time Interval}, \Delta t}$

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

directⁿ : Tangent to the path & is in the directⁿ of motion at any pt (at that pt)



$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}, \quad \tan\theta = \frac{v_y}{v_x}$$

ACCELERATION :-

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(v_x\hat{i} + v_y\hat{j})}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j}$$

MOTION IN A PLANE WITH CONSTANT ACCELERATION :-

Motion in a plane, can be treated as superposition of two separate simultaneous 1-D motions along two ⊥r directⁿs.

at $t=0$: $\vec{r}_0, \vec{v}_0$, at time, t : $\vec{r}, \vec{v}$, $\vec{a}$ = constant

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t} \Rightarrow \vec{v} = \vec{v}_0 + \vec{a}t$$

$$\therefore \quad v_x = v_{0x} + a_x t$$

cart. form $\quad v_y = v_{0y} + a_y t$

As, displacement = Average Velocity × time

$\therefore \quad \vec{r} - \vec{r}_0 = \left(\dfrac{\vec{v} + \vec{v}_0}{2}\right) t = \left[\dfrac{(\vec{v}_0 + \vec{a}t) + \vec{v}_0}{2}\right] t$

$\Rightarrow \vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

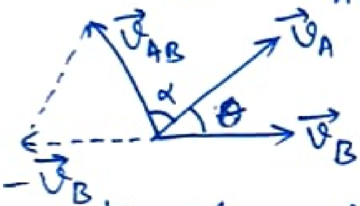
$\Rightarrow \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $\therefore$ comp. form: $\left\{ \begin{array}{l} x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{array} \right\}$

RELATIVE VELOCITY IN 2-D :-

Velocity of object A relative to that of B : $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Illy, Velocity of object, B relative to that of A : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$\therefore \quad \vec{v}_{AB} = -\vec{v}_{BA} \quad \& \quad |\vec{v}_{AB}| = |\vec{v}_{BA}|$

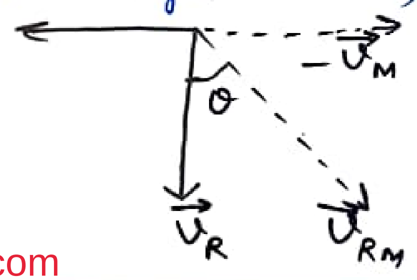


$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)}$

$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

$\tan\alpha = \dfrac{v_B \sin(180° - \theta)}{v_A + v_B\cos(180° - \theta)} = \dfrac{v_B \sin\theta}{v_A - v_B\cos\theta}$

Rain & Moving Man :- Man (west) Rain (Vertically downwards)



$|\vec{v}_{RM}| = \sqrt{v_R^2 + v_M^2 + 2v_R v_M \cos 90°}$

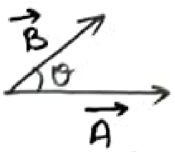
$= \sqrt{v_R^2 + v_M^2}$

& $\tan\theta = v_M / v_R$

PRODUCT OF VECTORS :- 1. Scalar Product (Dot Product), 2. Vector Product (Cross Product)

(1.) Scalar Product :- $\vec{A}\cdot\vec{B} = A\,\underline{B\cos\theta}$

$\llcorner$ component of $\vec{B}$ in the direct^n of $\vec{A}$.



If $\vec{A} \perp \vec{B}$: $\vec{A}\cdot\vec{B} = 0$ ($\because \cos 90° = 0$)

$$\boxed{\begin{array}{l} \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0 \end{array}}$$

(a) $\vec{A}\cdot\vec{A} = AA\cos 0° = A^2$

(b) Commutative : $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

(c) Distributive : $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

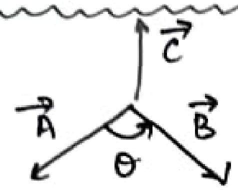
(d) Cartesian coord. :- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\Rightarrow \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$$\boxed{\begin{array}{l} \text{Examples:} \\ W = \vec{F}\cdot\vec{S} \\ P = \vec{F}\cdot\vec{v} \\ \phi = \vec{B}\cdot\vec{A} \end{array}}$$

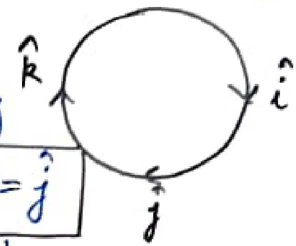
(2.) Vector Product :- $\vec{A} \times \vec{B} = \vec{C} = AB\sin\theta\, \hat{C}$



If $\vec{A} \parallel \vec{B}$: $\vec{A} \times \vec{B} = 0$ $(\because \sin 0^\circ = 0)$

(a) $\vec{A} \times \vec{A} = AA\sin 0^\circ = \underline{0}$

(b) Anticommutative : $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$



$$\boxed{\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}}$$

(C) Cartesian Coordinates :- $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Examples:
$\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{v} = \vec{\omega} \times \vec{r}$
$\vec{L} = \vec{r} \times \vec{P}$, $\vec{a} = \vec{\alpha} \times \vec{r}$

PROJECTILE MOTION :- An Object that's in flight after being thrown or projected is called a Projectile.

Trajectory: Path followed by a Projectile.

Combined effect of two Velocities :-

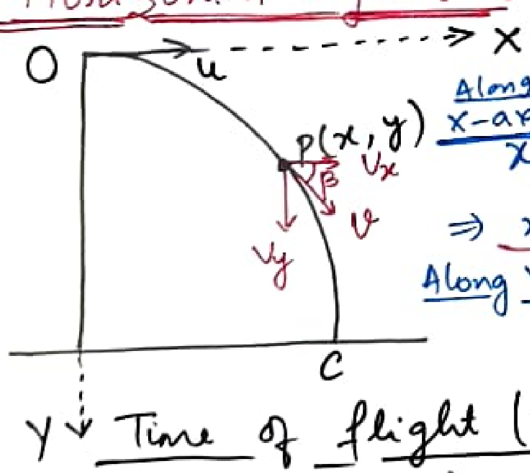
u_x : Horizontal (X) – constant comp.

u_x & u_y are indep. of each other.

u_y: Vertical component (Y) – Uniformly changing (↑ or ↓) due to g

Assumptions :- No frictional Air Resistance, No effect of Earth's Rotation, g – constant

Horizontal Projection :-



$(x_0, y_0) = (0, 0)$

Along x-axis: $u_x = u$, $u_y = 0$, $a_x = 0$, $a_y = g$

$x = x_0 + u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow \underline{x = ut}$ or $t = x/u$

Along Y-axis : $y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow \underline{y = \frac{1}{2}gt^2} = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \underline{kx^2}$ parabola

Time of flight (O→C) :- (T) $y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow h = \frac{1}{2}gT^2 \Rightarrow \boxed{T = \sqrt{\frac{2h}{g}}}$

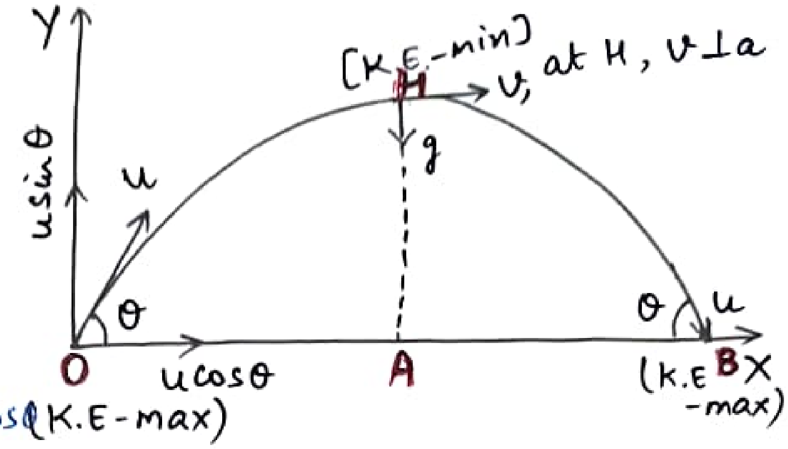
Horizontal Range (x=R) :- $x = x_0 + u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = 0 + u\sqrt{\frac{2h}{g}} + 0 \Rightarrow \boxed{R = u\sqrt{\frac{2h}{g}}}$

Velocity at any instant :- $V_x = u$, $V_y = u_y + a_y t = gt$

$\Rightarrow v = \sqrt{V_x^2 + V_y^2} = \sqrt{u^2 + g^2t^2}$ | $\tan\beta = \frac{V_y}{V_x} = \frac{gt}{u}$

Angular Projection :-



At $t=0$, $(x_0, y_0) = (0,0)$

$u \begin{cases} u\cos\theta \text{ (Uniform)} \\ u\sin\theta \end{cases}$

OB → due to $u\cos\theta$

AH → due to $u\sin\theta$

Along X-axis :- $u_x = u\cos\theta$ (K.E-max)

$a_x = 0$

$$x = x_0 + u_x t + \frac{1}{2}a_x t^2 \Rightarrow \underline{x = (u\cos\theta)t} \Rightarrow \underline{t = \frac{x}{u\cos\theta}}$$

Along Y-axis :- $a_y = -g$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2 \Rightarrow y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = x\tan\theta - \left(\frac{1}{2}\frac{g}{u^2\cos^2\theta}\right)x^2}$$ ← Eqn of the path of a Projectile.

Time of Flight :- T = Time of ascent + Time of descent

$= t + t = 2t$

at H: $u_y = u\sin\theta$, $v_y = 0$, $a_y = -g$, $t = T/2$

$v_y = u_y + a$

$$0 = u\sin\theta + (-g)T/2 \Rightarrow \boxed{T = \frac{2u\sin\theta}{g}}$$

Maximum Height :- $u_y = u\sin\theta$, $a_y = -g$, $y_0 = 0$, $y = H$, $t = T/2 = \frac{u\sin\theta}{g}$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$H = 0 + u\sin\theta \times \frac{u\sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$$

$$= \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\frac{u^2\sin^2\theta}{g}$$

$$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

Horizontal Range :- $R = (u\cos\theta)T = u\cos\theta \times \frac{2u\sin\theta}{g}$

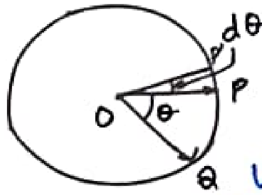
$$\Rightarrow \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

R_{max} — when $\sin 2\theta$: max $\Rightarrow 2\theta = 90° \Rightarrow \underline{\theta = 45°}$

$$\therefore \boxed{R_m = \frac{u^2}{g}}$$

CIRCULAR MOTION :-

Angular displacement $(\Delta\theta)$:- position of the particle, may be described by θ



$\theta = \frac{QP}{r}$

$[M^0L^0T^0]$

Unit - radian

$$\Delta\theta = \frac{PP'}{r} \Rightarrow \boxed{\Delta\theta = \frac{\Delta s}{r}}$$

Angular Velocity (ω) :- from P to P' : Object covers angular displacement of $\Delta\theta$ in time Δt

$[M^0L^0T^{-1}]$
rad s^{-1}

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Also, $\Delta s = r\,\Delta\theta$

$$\Rightarrow \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \Rightarrow \boxed{v = r\omega}$$

ω, T, ν :-
at $t = T$, $\theta = 2\pi$ rad
$$\omega = \frac{\theta}{T} = \frac{2\pi}{T} = 2\pi\nu$$

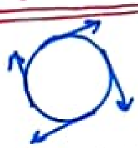
Angular Acceleration (α) : change in angular velocity is $\Delta\omega$ in time, Δt.

$[M^0L^0T^{-2}]$
rad s^{-2}

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

As, $a = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} \Rightarrow \boxed{a = r\alpha}$

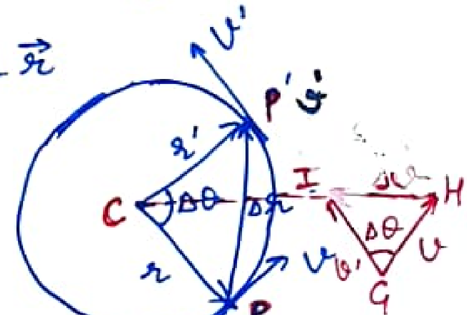
UNIFORM CIRCULAR MOTION :-



An Object is said to be in Uniform Circular Motion, if it follows a circular path at constant speed. As, the direction of the object changes continuously (at every pt), therefore, velocity changes & the object undergoes acceleration.

$\vec{r} \perp \vec{v}$ & $\vec{a} = \frac{\vec{v}}{t}$ $\therefore$ $\vec{a} \perp \vec{r}$ (Avg)

If we place $\Delta\vec{v}$ on the line that bisects the angle b/w $\vec{r}$ & $\vec{r}'$ we see that, it's directed towards the centre of the circle.



As, $\Delta t \to 0$, Avg acceleration b'comes the instantaneous acceleration & it's directed towards the centre of the circle.

$$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t}$$

$\Delta CPP' \approx \Delta GHI$
(formed by pos^n vector) (formed by velocity vector)

$$\Rightarrow \frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{R}$$

$$\Rightarrow |\Delta\vec{v}| = v\frac{|\Delta\vec{r}|}{R}$$

$$\therefore |\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v|\Delta\vec{r}|}{R \cdot \Delta t} = \frac{v}{R}\lim_{\Delta t \to 0}\frac{|\Delta\vec{r}|}{\Delta t}$$

If Δt is small, $\Delta\theta$ will also be small then arc PP' can be approximately taken to be $|\Delta\vec{r}|$:

$$|\Delta\vec{r}| \cong v\,\Delta t$$

$$\Rightarrow \frac{|\Delta\vec{r}|}{\Delta t} \cong v$$

$$\text{OR} \quad \lim_{\Delta t\to 0} \frac{|\Delta\vec{r}|}{\Delta t} = v$$

$\therefore$ Centripetal Acceleration, $a_c = \left(\frac{v}{R}\right)v = \frac{v^2}{R}$

$$\Rightarrow \boxed{a_c = \frac{v^2}{R} = \omega^2 R = 4\pi^2\nu^2 R}$$

$\{\because v = \omega R\}$

$\{\because \omega = 2\pi\nu\}$