SHORT NOTES: CLASS 11

CHAPTER 2: MOTION IN A STRAIGHT LINE

MOTION: An object is said to be in motion when its position changes with respect to time.
Nothing is in absolute rest or in absolute motion- motion is a combined property of an object under study and the observer. Motion is a relative term.
REFERENCE POINT- A fixed point or a fixed object w.r.t. which the given body changes its position is known as reference point.
Frames of reference can be of two types:
Inertial frames of reference are the one in which Newton's first law of motion is fully obeyed.
Non-Inertial frames of reference is the one in which Newton's first law of motion does not hold good.
Point Object- Size of the object < < Distance covered by the object.

PHYSICAL QUANTITIES

Physical Quantities- The quantities which can be measured are known as physical quantities.
Examples: length, mass, time, temperature, intensity, area, volume
Physical quantities are of two types: (i) Scalar Quantities (ii) Vector Quantities
Scalar Quantities- The physical quantities that are fully described by magnitude (numerical value) alone. Examples: Mass, Length, Time, Distance, Speed, Temperature
Vector Quantities- The physical quantities that are fully described by both the magnitude and direction. Examples: Displacement, Velocity, Acceleration, Force etc.

PATH LENGTH AND DISPLACEMENT

Path length- The length of the actual path traversed by an object is called distance/path length. it is a Scalar Quantity. Its value can never be zero or negative during its motion. S.I. Unit is m.
Displacement- The shortest distance measured from the initial position to final position of an object is called its displacement. It is a Vector Quantity. Its value can be positive, negative or zero. S.I. Unit of Displacement is m.

SPEED AND VELOCITY

Speed- Speed of an object is defined as the ratio of the total path length (i.e., actual distance travelled) in any direction in a unit time. i.e,

$$\text{Speed} = \frac{Total\ Path\ Length}{Time\ taken}$$

S.I. unit of speed is m/s. It is a scalar quantity. The speed of the object can be positive or zero but can never be negative.
Uniform speed: An object is said to be moving with uniform speed, if it covers equal distances in equal intervals of time.
Variable speed: An object is moving with variable speed if it covers unequal distances in equal intervals of time or equal distances in unequal intervals of time, howsoever small these intervals may be.
Average speed: Average speed for the given medium is defined as the ratio of the total distance travelled by the object to the total time taken.

$$\text{Average Speed} = \frac{Total\ Distance\ travelled}{Total\ Time\ taken}$$

- If a particle travel distances S_1, S_2, S_3.... etc with speeds v_1, v_2,v_3.....etc respectively, in the same direction.

$$\text{Average speed, } v_{av} = \frac{s_1+s_2+s_3+\cdots}{\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}+\cdots}$$

- If a body covers equal distance, S with different speeds v_1 and v_2. Then,

$$v_{av} = \frac{2v_1v_2}{v_1+v_2}$$

- If a body covers a distance, s. Let in time t_1, velocity is v_1, at time t_2, velocity is v_2 and so on...

$$d = v_1t_1 + v_2t_2 + v_{3t_3+\cdots}$$
$$v_{av} = \frac{v_1t_1 + v_2t_2 + \cdots}{t_1 + t_2 + \cdots}$$
If $t_1 = t_2 = \cdots = t_n = t$

Then, $\boldsymbol{v_{av}} = \frac{\boldsymbol{v_1+v_2+\cdots}}{\boldsymbol{n}}$

Instantaneous speed: The speed of an object at a given instant of time or at a particular point of its path is called its instantaneous speed.

$$\textbf{Instantaneous Speed} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

Instantaneous speed is measured by speedometer.

Velocity- Velocity of an object is defined as the distance travelled in a given direction in a unit time. i.e.,

$$\textbf{Velocity} = \frac{\textbf{Distance travelled (in a given direction)}}{\textbf{Time Taken}}$$

S.I. Unit of velocity is m/s. It is a Vector Quantity.

Velocity = Speed + Direction

Uniform velocity: When an object undergoes equal displacements in equal intervals of time. It is said to be moving with uniform velocity.

Variable velocity: An object is said to be moving with a variable velocity if either its speed changes with time or direction of motion changes with time or both changes with time.

Average velocity: When the object is moving with variable velocity, then average velocity of the object is defined as the ratio of total displacement to the total time interval in the given displacement.

$$\overrightarrow{v_{av}} = \frac{\textit{Total Displacement}}{\textit{Total time taken}}$$

If $\overrightarrow{x_1}$ and $\vec{x}_2$ are the displacements of the object in timings t_1 and t_2 respectively, then

$$\overrightarrow{v_{av}} = \frac{\vec{x}_2 - \vec{x}_1}{\vec{t}_2 - \vec{t}_1}$$

$$\textit{In uniform motion along a straight line, the average velocity is equal to uniform velocity}$$

Instantaneous Velocity, $\vec{v}_i$: Velocity of an object at an instant of time or at a particular point of its path is called Instantaneous velocity.

$$\vec{v}_i = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

$$\textit{In uniform motion along a straight line,}$$
$$\textit{the intantaneous velocity} = \textit{average velocity} = \textit{uniform velocity}$$

UNIFORM MOTION AND NON UNIFORM MOTION

Uniform Motion-

- An object is said to be in Uniform Motion, if it travels equal distance in equal intervals of time. However, small these intervals may be e.g., a vehicle running at constant speed.
- No Force is required for an object in Uniform Motion.
- The object moves with constant speed in Uniform Motion.
- The Distance-Time graph for an object in uniform motion is a straight line.

Non-Uniform Motion-

- An object is said to be in Non-Uniform Motion, if it travels unequal distance in equal intervals of time; the motion of a freely falling object is Non-Uniform Motion.
- The object moves with variable speed in Non-Uniform Motion.
- The Distance-Time graph for an object in Non-Uniform Motion is a curved line.

ACCELERATION

Acceleration- Acceleration of an object is defined as the rate of change of its velocity with time. Suppose, initial velocity of an object is ‘u’ and it attains a final velocity ‘v’ in time‘t’.

Then, **Acceleration, a = $\frac{\text{Final Velocity} - \text{Initial Velocity}}{\text{Time Taken}}$**

i.e, a = $\frac{v-u}{t}$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Its S.I. unit is m/s^2. It is a Vector Quantity.

- Retardation/Deceleration- Negative acceleration.
- For an object in uniform motion, acceleration is zero because, in this case, change in velocity (final velocity-initial velocity) is zero.

GRAPHICAL REPRESENTATION OF MOTION

A Graph is a pictorial representation of the relation between two sets of data of which one set is of dependent variables and the other set is of independent variables.

Quantities that can be calculated using graph: one by finding slope of the curve and other by finding area under curve.

Distance-Time Graph:

(i) For a stationary body- The graph is always a straight line parallel to time axis.

(ii) For Uniform Motion/Uniform Velocity- The slope of graph is a straight line.

(iii) For Non-Uniform Motion/Variable Velocity- Curved line.

(a) When the speed increases with time- increasing slope

(b) When the speed decreases with time- decreasing slope

Velocity-Time Graph:

(i) For the object moving with constant velocity- straight line parallel to time axis.

(ii) For Uniform Motion/Uniform Acceleration- The slope of graph is a straight line

(iii) For Non-Uniform Motion/ Variable Acceleration: the graph is a curve.

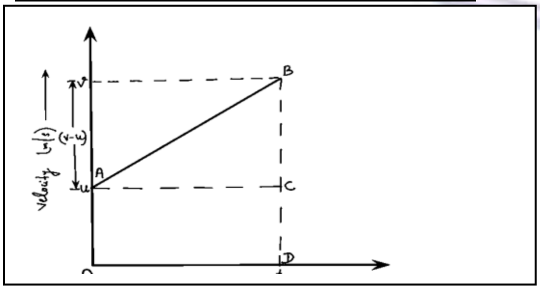
EQUATIONS OF MOTION

When a body moves along a straight line under uniform acceleration, the relation between its velocity, acceleration, distance covered & time taken can be found by equations known as "Equations of motion".

Ist Eqn of Motion	Velocity – Time Relation	$V = u + at$
2nd Eqn of Motion	Position – Time Relation	$S = ut + ½ at^2$
3rd Eqn of Motion	Position – Velocity Relation	$V^2 - u^2 = 2as$

Where, u = Initial Velocity, V = Final Velocity, t = time taken, s = distance covered, a = acceleration. Let, at time t= 0, the given object is moving uniformly with velocity, u and after time t, its velocity changes to let say v.

Derivations of Equations of Motion:



First Equation of Motion:

a = (v – u)/t;

v = u + at

From graph:

a = Slope of AB = BC/AC = (BD-CD)/AC

= (v-u)/t

a = (v-u)/t

v = u + at.

Second Equation of Motion:

S = Area under v-t graph

= ar (ABCDOA)

Third Equation of Motion:

S = Area under v-t graph

= ar (trapezium ABDO)

= ar (□ ACDO) + ar (Δ ABC)
= OA. OD + ½ AC. BC
=ut + ½ t(v-u)
=ut + ½ t.at
=ut + ½ at^2

= ½ (v+u) t
= ½ (v+u) {(v-u) /a}
= ½ $(v^2-u^2)/a$
=> $v^2-u^2 = 2as$

CALCULUS METHOD FOR DERIVING EQUATIONS OF MOTION:

At t = 0, velocity of the object = u and at time t, velocity = v. distance covered is s and acceleration = a

First Equation of Motion:

As, a = $\frac{dv}{dt}$

$\int_u^v \; v \cdot dv = \int_0^t a\,dt$ (v- u)= a (t-0) v = u + at

Second Equation of Motion:

$$v = \frac{ds}{dt}$$

$$ds = v\,dt$$
$$ds = (u + at)\,dt$$

$$\int_0^S dS = \int_0^t (u + at)\,dt$$

$$[s]_0^s = [ut]_0^t + \frac{1}{2}a[t^2]_0^t$$

$$s = ut + \frac{1}{2}at^2$$

Third Equation of motion: $a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$

$$\left[\because \frac{ds}{dt} = \text{Rate of change of displacement} = \text{v}\right]$$

$$\therefore ads = vdv$$

Integrating both sides within the conditions, we get a$\int_0^s ds = \int_u^v vdv$

$$\therefore as = \left[\frac{v^2}{2}\right]_u^v$$

$$2as = v^2 - u^2 \;\; or\; v^2 - u^2 = 2as$$

In vector from this equation can be written as $\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$.

DISTANCE TRAVELLED BY AN OBJECT IN nth SECOND:

Distance travelled in n seconds= s_n = un + ½ an^2

Distance travelled in (n-1) seconds= s_{n-1} = u(n-1) + ½ $a(n-1)^2$

Distance travelled in nth second= s_n - s_{n-1}

= (un + ½ an^2) – {u(n-1) + ½ $a(n-1)^2$}
= un + ½ an^2 -un + u - ½ {a(n^2 + 1 – 2n)}
= u + ½ an^2 - ½ an^2 - $\frac{a}{2}$ + an
= u + $\frac{a}{2}$(2n – 1)

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$